

# Canonical Representation of Manipulator Dynamics

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**Abstract** — A structural analysis of the matrix form of Lagrangian function, presenting the dynamic model of a controllable n-links mechanical system, is made. The linear dependence or independence of columns of coefficients in the matrix regression model of the dynamics gives the basis for eliminating or combining the classical mass-inertial parameters of the system. As a result of the structural analysis, a canonical dynamic model is proposed. The model ensures the minimum number of computation operations for the purpose of studying the overall dynamic behavior of an n-links manipulation robot.

**Keywords** — Manipulation Robots, Canonical Dynamic Model, Structural Analysis, Linearly Dependent Matrix Columns.

## I. INTRODUCTION

Contemporary control methods of manipulation robots require knowledge of its full dynamic model. The dynamic model includes geometric parameters of the manipulator links, inertial parameters of these links, coefficients of friction and dynamic parameters of the drive system. There are well developed methods for determining the geometric parameters of manipulation robots as well as procedures to determine the coefficients of friction in the mechanical part and the drive system.

The major problem in the study of the dynamic model is the precise definition of inertial parameters of the links and the manipulator, as a whole object. The dynamics of each link of a manipulator is given with its mass-inertial parameters, expressed in local coordinate systems.

Namely, they are:  $m_i$  - the mass of the  $i$  link,  $\{m_i \bar{x}_i, m_i \bar{y}_i, m_i \bar{z}_i\}$  - vector of coordinates of the center of gravity of the  $i$  link,  $\{I_{xxi}, I_{xyi}, I_{xzi}, I_{yyi}, I_{yzi}, I_{zzi}\}$  - components of the tensor of inertia of the link  $i$ .

Therefore, the full number of inertial parameters of a link  $i$  is 10, and for a manipulator composed of  $n$  series connected links is  $10n$ .

## II. LAGRANGIAN DYNAMIC MODEL

When  $n$  number solids links are connected, a new phenomenon arises related to the dynamics of the mechanical chain, as a whole. Due to geometric and kinematic constraints in the mechanical chain, each of the above mentioned parameters have varying degrees of impact on the dynamic behavior of the manipulator, i.e. on the

formation of generalized forces and moments, driving the whole mechanical system. The whole set of  $10n$  inertial parameters for a manipulation robot is broken down into three sub-sets, namely:

- Sub-set of inertial parameters which do not affect the dynamics,
- Sub-set of inertial parameters which have independent effects on the dynamics,
- Sub-set of inertial parameters which influence the dynamics in the form of linear combinations.

Therefore, before to begin finding numerical values of an inertial parameter it is appropriate to make a partition of the set of inertial parameters in the aforementioned sense. The partition of the inertial parameters is based on the dynamic model, using the formalism of Lagrange, namely [1]:

$$\frac{d}{dt} \left( \frac{\partial L}{\partial \dot{\mathbf{q}}} \right) - \frac{\partial L}{\partial \mathbf{q}} = \boldsymbol{\tau} \quad (1)$$

$i = 1, 2, \dots, n$ ,

where  $L(\mathbf{q}, \dot{\mathbf{q}}) = K(\mathbf{q}, \dot{\mathbf{q}}) - P(\mathbf{q})$ ,

$K(\mathbf{q}, \dot{\mathbf{q}})$  and  $P(\mathbf{q})$  are the kinetic and the potential energy of the system.

After replacing the relevant expressions for the kinetic and the potential energy of the mechanical system according to equation (1), one can come to the well known equation of the dynamics of n-link manipulation robot [2]:

$$\mathbf{D}(\mathbf{q}, \boldsymbol{\Theta}) \ddot{\mathbf{q}} + \mathbf{H}(\mathbf{q}, \boldsymbol{\Theta}) \dot{\mathbf{q}} \dot{\mathbf{q}} + \mathbf{G}(\mathbf{q}, \boldsymbol{\Theta}) = \boldsymbol{\tau} \quad (2)$$

$\boldsymbol{\Theta} = f(m_i, m_i \bar{x}_i, m_i \bar{y}_i, m_i \bar{z}_i, I_{xxi}, I_{xyi}, I_{xzi}, I_{yyi}, I_{yzi}, I_{zzi})$

where

$\boldsymbol{\Theta}$  - is a vector-function of the classical mass-inertial

parameters of the links.  $\mathbf{q} = [q_1, q_2, \dots, q_n]^T$  is  $(n \times 1)$

general coordinates vector,

$\dot{\mathbf{q}} \dot{\mathbf{q}} = [\dot{q}_1 \dot{q}_1 \dots \dot{q}_1 \dot{q}_n \dots \dot{q}_2 \dot{q}_2 \dots \dot{q}_{n-1} \dot{q}_n]^T$  general velocities

combinations vector,  $\ddot{\mathbf{q}} = [\ddot{q}_1, \ddot{q}_2, \dots, \ddot{q}_n]^T$  is  $(n \times 1)$

general accelerations vector,  $\boldsymbol{\tau}$   $(n \times 1)$  vector of driving torques,

$\mathbf{D}(\bullet)$   $(n \times n)$  is a symmetric inertial matrix,  $\mathbf{H}(\bullet)$  is  $(n \times n(n-1)/2)$  is the matrix of coefficients of

Coriolis and centrifugal forces,  $\mathbf{G}(\bullet)$  is  $(n \times 1)$  is the vector of coefficients of gravity forces.

$\mathbf{D}(\bullet), \mathbf{H}(\bullet), \mathbf{G}(\bullet)$  are matrix coefficients for the equation (2) and are functions of kinematic and inertial parameters of the manipulator.

### III. THE CANONICAL DYNAMIC MODEL

The partition of dynamic parameters  $\Theta$  is done on the basis of the equation (2) after its transformation into a canonical form. The Lagrange function  $L(\mathbf{q}, \dot{\mathbf{q}})$  of n-links manipulator can be written as a sum:

$$L(\mathbf{q}, \dot{\mathbf{q}}) = \sum_{i=1}^n L_i(\mathbf{q}, \dot{\mathbf{q}}) \quad (3)$$

In turn, the Lagrange function  $L_i(\mathbf{q}, \dot{\mathbf{q}})$  for each link  $i$  is regrouped based on the following rule: for each link is formed a function  $\hat{L}_i$  as a sum of those elements of (3) that depend on at least one  $\{q_i, \dot{q}_i\}$ , but do not depend on any one  $\{q_k, \dot{q}_k\}$ ,  $k = i+1, \dots, n$ , i.e. each function  $\hat{L}_i$  depends on the position and/or velocity of all links to the link  $i$ , but does not depend on the position and/or velocity of the links after the link  $i$ . This rearrangement procedure is followed in the following way [3]:

$$\sum_{i=1}^n \sum_{j=1}^{p_i} \hat{\tau}_{ij}(q_1 \dot{q}_1, \dots, q_i \dot{q}_i) \hat{X}_{ij}(\Theta_i) \quad (4)$$

where  $\hat{\tau}_{ij}$  is a function depending only on kinematic parameters,  $\hat{X}_{ij}$  is a function only on inertial parameters,

$p_i$  is the number of terms in (4) for each  $\hat{L}_i$ .

After replacing the expression (4) in the expression (1) and carrying out appropriate actions, the result leads to the form of the dynamic model:

$$D_{ij}(\mathbf{q}, \dot{\mathbf{q}}, \ddot{\mathbf{q}}) \hat{X}_{ij}(\Theta_i) = \tau_i \quad (5)$$

After additional appropriate rearrangement of (5), the following structure is achieved:

$$\begin{bmatrix} \tau_1 \\ \vdots \\ \tau_n \end{bmatrix} = \begin{bmatrix} D_{11} & \dots & D_{1M} \\ \dots & \dots & \dots \\ D_{n1} & \dots & D_{nM} \end{bmatrix} \begin{bmatrix} \hat{X}_1 \\ \vdots \\ \hat{X}_M \end{bmatrix} = \quad (6)$$

$$D^1 \hat{X}_1 + \dots + D^m \hat{X}_m + \dots + D^M \hat{X}_M = \tau^1 + \dots \tau^m + \dots \tau^M$$

where  $m$  is the inertia parameter after rearrangement,

$D^m$  is part of the dynamic coefficient  $D$ , which is influenced by the inertia parameter  $\hat{X}_m$ , and  $\tau^m$  is the part of the general moment on which the inertial parameter  $\hat{X}_m$  influences.

Equation (6) is the canonical dynamic model used to regroup the whole set of inertial parameters for a given mechanical configuration.

### IV. REGROUPING PROCEDURE

As a result of specific kinematic structure of a given manipulation system, some of inertial parameters do not

affect the dynamic behavior, while others have combined influence. These features are used to minimize the number of identifiable inertial parameters. The minimization is achieved based on the following rules:

1  $\rightarrow$  the inertial parameter  $X_{ij}$  has no effect on the dynamic

if vector  $D_{ij}$  in (5) is zero.

2  $\rightarrow$  the inertial parameter  $X_{ij}$  can be substituted by other

inertial parameters  $\{X_{i1}, X_{i2}, \dots, X_{ir}\}$  if the vector  $\mathbf{D}^i$  is linearly dependent of column vectors  $\{\mathbf{D}^{i1}, \mathbf{D}^{i2}, \dots, \mathbf{D}^{ir}\}$ , that means, if it satisfies the following:

$$\mathbf{D}^i = \alpha_{i1} \mathbf{D}^{i1} + \dots \alpha_{ik} \mathbf{D}^{ik} + \dots \alpha_{ir} \mathbf{D}^{ir} \quad (7)$$

where  $\alpha_{ik}$  is a constant,  $k = 1, 2, \dots, r$ .

Regrouping of the inertial parameters is carried out by a procedure, leading to the removal of  $\mathbf{X}^i$  and  $\mathbf{D}^i$  in (5). This is achieved by replacing the parameter  $X_{ik}$  by an expression  $X_{ikr}$ , that has the form:

$$X_{ikr} = X_{ik} + \alpha_{ik} X_i \quad (8)$$

The advantage of regrouping or/and elimination of some inertial parameters obviously leads to a reduction of the complexity of the dynamic model, and in particular leads to excluding the calculations, associated with the column  $\mathbf{D}^i$ .

### V. CONCLUSION

Based on analysis of the impact of the classical mass-inertial parameters for a system of interconnected and controllable solid links, a canonical dynamic model is proposed. The advantage of the canonical model is that it provides a minimum number of computing operations for the purpose of studying of overall dynamic behavior of n-links manipulation robot.

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