International Journal of Applied Science and Mathematics
Volume 3, Issue 1, ISSN (Online): 2394-2894

Hydromagnetic Boundary Layer Slip Flow and Heat Transfer with Thermal Radiation and Viscous Dissipation

Christopher Munyiwa Kaneba
Department of Pure and Applied Mathematics
Kenyatta University
chriskaneba@gmail.com

Winifred Nduku Mutuku
Department of Pure and Applied Mathematics
Kenyatta University
mutukuwinnie@gmail.com

***Abstract* – The simultaneous effects of radiation and viscous dissipation on heat transfer over a moving sheet with slip in both velocity and boundary condition has been numerically analyzed. The governing boundary layer equations are formulated and transformed into ordinary differential equations using similarity transformation. The resulting ordinary differential equations are solved numerically using the Runge-Kutta fourth order algorithm coupled with a shooting technique. Numerical computations are performed for pertinent values of the physical parameters involved. For illustration of the results, numerical values are plotted in graphical form and discussed quantitatively with respect to the influence of governing physical parameters, namely, magnetic field parameter M, Eckert number Ec, Biot Number Bi, Injection/Suction parameter f_w, Radiation Parameter R and slip parameter β. The Prandtl number of the based fluid (water) is kept constant at 6.2. It is noted that the magnetic parameter slackens the fluid motion whilst increasing it temperature and skin friction coefficient.**

***Keywords* – Boundary Layer, Heat Transfer, Radiation, Slip Flow, Viscous Dissipation.**

I. Introduction

The problem on boundary layer flow of an electrically conducting fluid in the presence of magnetic field has a varied range of applications in many engineering problems namely; MHD generators, in the polymer industry, nuclear reactors, geothermal energy extraction among others. The flow and radiative heat transfer characteristics induced by a continuously moving surface are of a great significance in many engineering applications. This is because the quality of the final product depends on the rate of heat transfer of the ambient fluid particles and the magnetic field intensity. Experimental and theoretical investigations on conventional electrically conducting fluids indicate that magnetic field changes the transport and heat characteristics of conventional fluids. The problem on steady boundary layer flow due to a continuously moving surface was first studied by Sakiadis [1]. Thereafter, several authors [2]-[6] extended the study of the same problem for various aspects for both Newtonian and non-Newtonian fluids.

The effect of thermal radiation on heat transfer processes is very significant at high operating temperatures and as a result it has many engineering applications. In recent times, many researchers have included the effect of thermal radiation in their work. Makinde [7] studied the effect of thermal radiation and mass transfer past a moving vertical porous plate. Sajid and Hayat [8] showed the influence of thermal radiation on the flow over an exponentially stretching sheet and they solved the problem analytically using the homotopy analysis method. K Das [9] examined the impact of thermal radiation on MHD slip flow over a flat plate with variable fluid properties. Motsumi and Makinde. [10] studied the effects of thermal radiation and viscous dissipation on boundary layer flow of nanofluids over a permeable moving flat plate .Several other authors [11]-[14] have studied the effect of thermal radiation under different conditions. The main purpose of the present study is to investigate the combined effects of hydro magnetic boundary layer slip flow and heat transfer with thermal radiation and viscous dissipation on a horizontal moving plate surface. In subsequent sections of this paper we will have a formulation of the model governing equations, and then we will reduce the model governing equations to a dimensionless form. Thereafter we will use a Maple computer programme to get our results which is presented in graphical and tabular form. We then have a discussion where the graphical representation of the pertinent parameters on the flow field and heat transfer characteristics is displayed and discussed in detail.

II. Mathematical Formulation of the Problem

Consider the steady laminar incompressible two dimensional boundary layer flow of an electrically conducting fluid flowing past a highly lubricated surface. We choose the coordinate system such that x-axis is along the horizontal plate and y-axis is orthogonal to the plate. The physical flow model and coordinate system is shown in Fig. 1 below. A transverse magnetic field of strength B_0 is applied parallel to the y-axis. The variable plate surface permeability function is defined by $V_0 = -f_w(av_f)^{1/2}$ where f_w is the suction/injection parameter; with $f_w > 0$ representing the transpiration rate at the plate surface, $f_w < 0$ corresponds to injection and $f_w = 0$ for an impermeable surface.

Considering the fluid as a continuous media with thermal equilibrium and slip occurring between the base fluid and the solid particles, the basic steady governing

International Journal of Applied Science and Mathematics
Volume 3, Issue 1, ISSN (Online): 2394-2894

equations of continuity, momentum and thermal energy are given respectively as;

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=-\frac{1}{\rho_f}\frac{\partial p}{\partial x}+\upsilon\frac{\partial^2 u}{\partial y^2}-\frac{\sigma B_0^2}{\rho_f} \tag{2}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\frac{\partial^2 T}{\partial y^2}+\frac{\mu}{(\rho C_p)_f}\left[\frac{\partial u}{\partial y}\right]^2+\frac{\sigma B_0^2 u^2}{(\rho C_p)_f}-\frac{1}{(\rho C_p)_f}\frac{\partial q_r}{\partial y} \tag{3}$$

The boundary conditions are given as

$$\lambda u=\mu_f\frac{\partial u}{\partial y}, V=V_w,\ -k_f\frac{\partial T}{\partial y}=h_f(T_f-T)$$

At $y=0$ (4)

$u=u_\infty$, $T=T_\infty$, at $y=\infty$

Where u and v are the velocity components along the x and y directions respectively, T_f is temperature, T_∞ is the free stream temperature, $u_\infty = ax$ is the free stream velocity which implies that the free stream velocity is increasing with axial distance along the plate surface, ρ is the density, $\alpha = k/(\rho C_p)_f$ is the thermal diffusivity, k is the thermal conductivity, $\upsilon = \mu/\rho_f$ is the kinematic viscosity coefficient, μ is the dynamic viscosity, C_p is the specific heat at constant pressure, σ is the electrical conductivity.

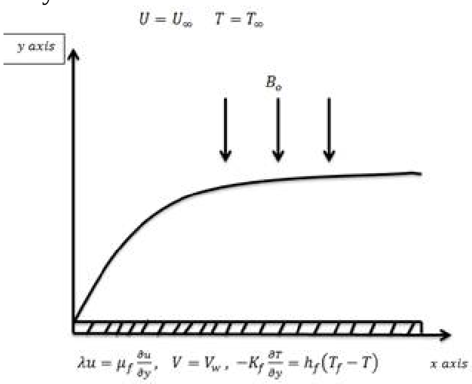

Fig. 1. Schematic Diagram of Flow

Using the Roseland approximation for the thermal radiation [15], the radiative heat flux is simplified as;

$$q_r=-\frac{4\sigma^*}{3k^*}\frac{\partial T^4}{\partial y} \tag{5}$$

Where σ^* is the Stephan-Boltzmann and k^* is the mean absorption coefficient.

The temperature differences within the flow are assumed to be sufficiently small such that T^4 may be expressed as a linear function of temperature T. This is done by expanding T^4 in a Taylor series about a free stream temperature T_∞ as shown below;

$$T^4=T_\infty^4+4T_\infty^3(T-T_\infty)+6T_\infty^2(T-T_\infty)^2\pm--$$

If we neglect higher order terms in the above equation beyond the first degree of $(T-T_\infty)$, we get

$$T^4\cong 4T_\infty^3 T-3T_\infty^4 \tag{6}$$

Substitute equation (4.6) into equation (4.5), we get

$$q_r=-\frac{16T_\infty^3\sigma^*}{3k^*}\frac{\partial T}{\partial y} \tag{7}$$

Using the boundary layer approximations and introducing equation (4.7), the governing equations become;

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{8}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=U_\infty\frac{\partial u_\infty}{\partial x}+\frac{\mu}{\rho}\frac{\partial^2 u}{\partial y^2}-\frac{\sigma B_0^2(U-U_\infty)^2}{\rho_f} \tag{9}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{k}{\rho C_p}\frac{\partial^2 T}{\partial y^2}+\frac{\mu}{(\rho C_p)_f}\left[\frac{\partial u}{\partial y}\right]^2+\frac{\sigma B_0^2(U-U_\infty)^2}{(\rho C_p)_f}-\frac{16\sigma^* T_\infty^3}{3k}\frac{\partial^2 T}{\partial y^2} \tag{10}$$

Using the stream function $\psi=\psi(x,y)$ the velocity components u and v are defined as;

$$u=\frac{\partial\psi}{\partial y}\ \text{and}\ v=-\frac{\partial\psi}{\partial x} \tag{11}$$

By means of similarity transformations, the dimensionless variables are defined as

$$\eta=(a/\upsilon_f)^{1/2},\ \psi=(a\upsilon_f)^{1/2},\ \theta(\eta)=\frac{T-T_\infty}{T_f-T_\infty} \tag{12}$$

The governing equations (8), (9), and (10) combined with the boundary equations are transformed into ordinary differential equations as follows:

$$f^{III}+ff^{I}-(f^{I})^2+1-M(f^{I}-1)=0 \tag{13}$$

$$\frac{1}{\Pr}(1+\frac{4}{3}R)\theta^{II}+f\theta^{I}+Ecf^{II^2}+HaEc(f^{I}-1)^2=0 \tag{14}$$

The corresponding boundary conditions are:

$$f^{I}(0)=\beta f^{II}(0),\ f(0)=f_w\ \theta^{I}=Bi[\theta-1],$$
$$f^{I}(\infty)=1,\ \theta(\infty)=0 \tag{15}$$

f^{I} and θ are the dimensionless velocity and temperature respectively. Pr, R, Ec and Ha denote Prandtl number, Radiation parameter, Eckert number and the Hartmann number also known as the magnetic parameter respectively

They are defined as

$$\Pr=\frac{v}{\alpha}\ R=\frac{4\delta^* T_\infty^3}{k^* k}\ Ec=\frac{\upsilon_f^2}{C_p(T_f-T_\infty)},\ Ha=\frac{\sigma B_0^2}{\rho_f a} \tag{16}$$

The physical quantities of practical significance in this dissertation are skin friction coefficient and Nusselt number. The skin friction coefficient C_f and the local Nusselt number Nu_x are expressed as:

International Journal of Applied Science and Mathematics
Volume 3, Issue 1, ISSN (Online): 2394-2894

$$C_f = \frac{\tau_w}{\rho_f U_\infty^2} \quad Nu = \frac{x q_w}{k_f (T_f - T_\infty)} \qquad (17)$$

Where τ_w is the wall shear stress and q_w is the heat flux from the plate. They are defined as:

$$\tau_w = \mu \frac{\delta u}{\delta y} \text{ as } y = 0 \text{ and } q_w = -k \frac{\delta T}{\delta y} \text{ as } y = 0 \qquad (18)$$

From above we get that

$Re_x^{1/2} C_f = f^{II}(0)$ and $Nur = Re_x^{-1/2} Nu_x = -\theta^I(0)$ Where

C_f is the local skin friction

Nur is the reduced Nusselt number

Re_x is the local Reynolds number

III. Solution of the Problem

The set of non-linear differential equations (13) and (14) with the boundary conditions (15) were solved numerically using Runge-Kutta fourth order algorithm with a systematic guessing of shooting technique until the boundary condition at infinity were satisfied. The computation was done by a computer programme which uses a symbolic and computational language Maple. Numerical computations were performed for various values of the physical parameters involved. Namely; Magnetic parameter M (Hartmann number Ha), Prandtl number $\Pr$ and Eckert number Ec . For illustration of the results, numerical values were plotted in figures 2 to 9 and a detailed discussion on the effects of the governing physical parameters on the velocity profile, temperature profile and skin friction coefficient was later done. The Prandtl number of the based fluid (water) was kept constant at 6.2.

IV. Results And Discussion

4.1 Effect of Parameter Variation on the Velocity Profile

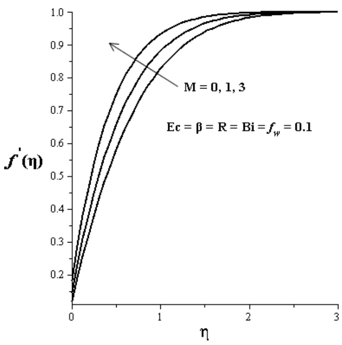

Fig. 2. Velocity profile against η with increasing Magnetic field intensity

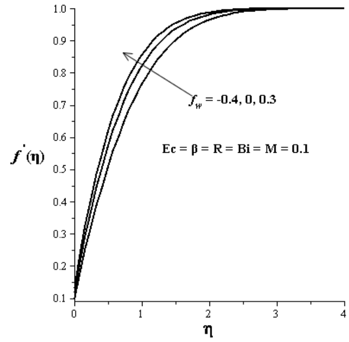

Fig. 3. Velocity profile against η with increasing suction/injection parameter

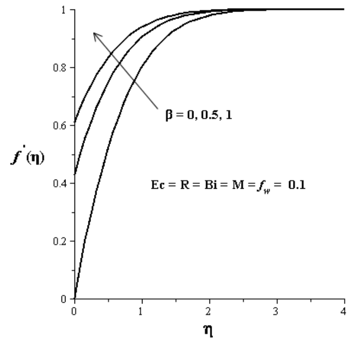

Fig. 4. Velocity profile with increasing Slip Parameter

Fig. 2 above shows the effects of different values of the magnetic parameter (M=0, 1, 3) on the velocity profile. It shows that velocity increases with increasing values of M. It is also noted that the thickness of the momentum boundary layer decreases. This is as a result of the Lorentz force which arises from the interaction of magnetic and electric fields during the motion of the electrically conducting fluid.

Fig. 3 above shows the effect of different values of the suction parameter $f_w = -0.4, 0, 0.3$ on the velocity profile. It shows that for a fixed value of η, the velocity profile increases as the suction parameter increases and as a result the boundary layer thickness becomes thinner.

Fig. 4 above shows the effects of different values of the slip parameter on the velocity profile. It is noted that the velocity increases with increasing values of the slip parameter and the momentum boundary layer decreases and becomes thinner.

4.2 Effect of parameter variation on the Temperature Profile

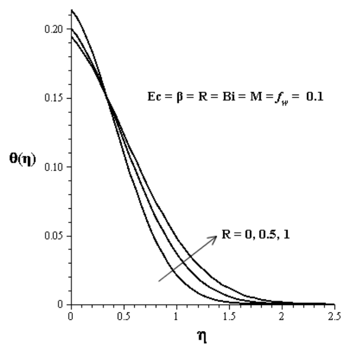

Fig. 5. Temperature profile with increasing Radiation parameter

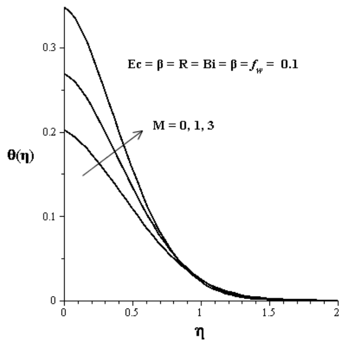

Fig. 6. Temperature profile with increasing Magnetic field intensity

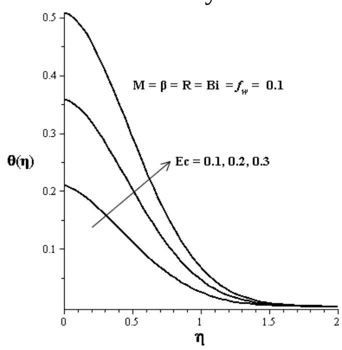

Fig. 7. Temperature profile with increasing Eckert number

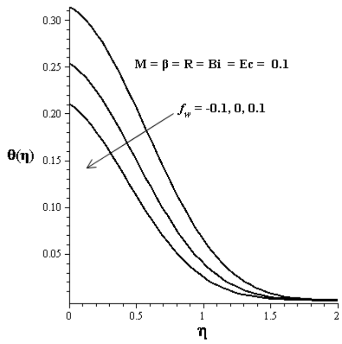

Fig. 8. Temperature profile with increasing Suction/injection parameter

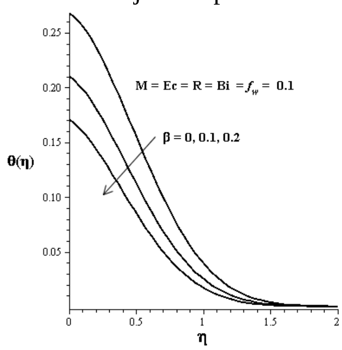

Fig. 9. Temperature profile with increasing Slip parameter

Fig. 5-9 above shows the effect of different parameters on the fluid temperature profile. It is noted that the temperature is maximum at the plate surface due to convectional heating but decreases exponentially to zero far away from the plate surface satisfying the free stream conditions. Fig 4 shows the effect of the radiation parameter on the temperature profile. Figure 5 shows the effect of different values of M on the temperature distribution. It is noted that temperature decreases with increasing values of M. The temperature field for various values of Suction/Injection parameter f_w is shown in fig. 7. It is noted that for a fixed value of $\boldsymbol{\eta}$, temperature decreases as f_w decreases. An increase in the Suction parameter means that more fluid is sucked through the permeable surface to the surrounding, hence a reduction in thermal boundary layer thickness is expected. Fig. 6 shows the effects of different values of Eckert number Ec on temperature. It is observed that for any fixed value of $\boldsymbol{\eta}$ temperature decreases as Ec increases. Fig 8 shows the effects of different values of the Slip parameter on the temperature profile.

4.3 Effects of Parameters Variation on the Skin Friction and Nusselt Number.

f_w	M	β	R	Ec	Bi	$f''(0)$	$-\theta'(0)$
0	0.1	0.1	0.1	0.1	0.1	1.1651	0.0355
1						1.6531	0.0814
2						2.1632	0.08876
3						2.6541	0.0914
0.1	**0**					1.1809	0.0745
	1					1.4431	0.0630
	2					1.6469	0.0542
	5					1.8163	0.0468
		0				1.3301	0.0445
		1				1.6085	0.0922
		2				1.3812	0.0947
		3				0.2767	0.0953
			0			1.3301	0.0617
			1			1.3301	0.0693
			2			1.3301	0.0719
			3			1.3301	0.3784
				0		1.3301	0.0945
				0.1		1.3301	0.0724
				0.2		1.3301	0.0503
				0.3		1.3301	0.0281

Table 1 above shows the effect of various governing parameters on the Skin friction coefficient $f^{II}(0)$ and Nusselt number $-\theta^{I}(0)$. It is seen that increasing f_w, increases both $f^{II}(0)$ and $-\theta^{I}(0)$. An increase in M leads to an increase in $f^{II}(0)$ but decreases the Nusselt number $-\theta^{I}(0)$. An increase in β reduces $-f^{II}(0)$ but it increases $-\theta^{I}(0)$. An increase in the Eckert number Ec has no effect on $-f^{II}(0)$ but it reduces $-\theta^{I}(0)$. An increase in R has no effect on $-f^{II}(0)$ but it increases $-\theta^{I}(0)$.

V. Conclusion

From the study we have shown that it is true that a higher Prandtl number has the effect of having a relatively lower thermal conductivity in a fluid which reduces conduction and thereby increasing heat transfer rate at the surface. The Prandtl number controls the relative thickness of the momentum and thermal boundary layers. When Prandtl number is small, it means that the heat diffuses quickly compared to the velocity (momentum). This means that for liquid metals the thickness of the thermal boundary layer is much bigger than the velocity boundary layer.

Abbreviations and Acronyms

(u, v)	Velocity components
(x, y)	Coordinates
B_0	Constant applied magnetic field
C_p	Specific heat at constant pressure
Nu	Local Nusselt number
Pr	Prandtl number
Ec	Eckert number
M	Magnetic parameter
q_w	Dimensional heat flux
Re_x	Local Reynolds number
T	Temperature
T_∞	Free stream temperature
U_∞	Free stream velocity
f_w	Suction/Injection parameter
k	Thermal conductivity
h_f	Heat transfer coefficient
T_w	Skin friction or shear stress

Greek symbols

ψ	Stream function
θ	Dimensionless temperature
η	Similarity variable
β	Thermal expansion coefficient
α	Thermal diffusivity
φ	Solid volume fraction
μ	Dynamic viscosity
υ	Kinematic viscosity
ρ	Density
λ	Slip coefficient

Subscripts

f	fluid
s	solid

References

[1] Sakiadis B.C.“Boundary layer behavior on continuous solid surfaces”. AIChE J 1961; 7:221–225.

[2] MY Hussaini, WD Lakin, A Nachman. “On similarity solutions of a boundary layer problem with an upstream moving wall”. SIAM J Appl Math 1987; 47:699–709.

[3] N Afzal, IS Varshney. “The cooling of a low heat resistance stretching sheet moving through a fluid”.Wa rme-und Stoffu Bertragung 1980;14:289–293.

[4] FK Tsou, EM Sparrow, RJ Goldstein. “Flow and heat transfer in the boundary layer on a continuous moving surface”. International Journal for Heat Mass and Transfer 1967; 10:219–235.

[5] N Afzal, A Badaruddin, AA Elgarvi. “Momentum and transport on a continuous flat surface moving in a parallel stream”. International Journal for Heat Mass and Transfer 1993; 36:3399–4403.

[6] A Ishak, R Nazar, Pop I. “Boundary-layer flow of a micropolar fluid on a continuous moving or fixed surface”. Can J Phys 2006; 84:399–410.

[7] OD Makinde. “Free convection flow with thermal radiation and mass transfer past a moving vertical porous plate”. International Communication Heat Mass Transfer 2005; 32:1411–1419.

[8] M Sajid, T Hayat. “Influence of thermal radiation on the boundary layer flow due to an exponentially stretching sheet”. International Communication Heat Mass Transfer 2008;35:347–356

[9] K Das.“Impact of thermal radiation onMHD slip flow over a flate plate with variable fluid properties”. Heat Mass Transfer 2011; 48(5):767–778.

[10] T. G. Motsumi and O. D. Makinde. “Effects of thermal radiation and viscous dissipation on boundary layer flow of nanofluids over a permeable moving flat plate”.Phys.Scr.vol. 86, pp. 1-8, 2012.

[11] A Raptis. “Radiation and free convection flow through a porous medium”. International Communication Heat Mass Transfer 1998; 25:289–295.

[12] FS Ibrahim, AM Elaiw, AA Bakr. “Influence of viscous dissipation and radiation on unsteady MHD mixed convection flow of micropolar fluids”. Applied Math Inform Science 2008; 2:143–162.

[13] T Hayat, Z Abbas, I Pop, S Asghar. Effects of radiation and magnetic field on the mixed convection stagnation-point flow over a vertical stretching sheet in a porous medium. International J Heat Mass Transfer 2010; 53:466–474.

[14] W N Mutuku, “MHD boundary layer flow and heat transfer of nanofluids past a permeable moving flat surface with thermal radiation and viscous dissipation”. Universal Journal of Fluid Mechanics 2 (2014), 55-68

[15] Brewster MQ. *Thermal radiative transfer and properties*. Wiley

Author's Profile

Christopher Munyiwa Kaneba

Christopher Munyiwa Kaneba joined Jomo Kenyatta University of Agriculture and Technology and pursued Bsc Mathematics (Statistics and Applied Mathematics) from 2007 to 2011. He graduated with Second class honors upper division. He later joined Kenyatta University for Msc Applied Mathematics from 2013 to 2015 and graduated on December 2015 with a project in Computational Fluid Dynamics. He has previously worked as a high school teacher teaching Mathematics and Physics. He later joined the public service and worked with the Ministry of Devolution and Planning in Kenya for three years from 2012 to 2015. At the moment he is working as a Graduate Assistant in Maasai Mara University, Department of Mathematics and Physical Sciences. Mr. Kaneba has research interests in Mathematical Modeling and Complex Analysis of Systems. He has attended and presented at several International conferences and workshops on Mathematical Modeling.

Winifred Nduku Mutuku

Winifred Nduku Mutuku joined Kenyatta University and pursued Bachelor of Education (Science) from 1992 to 1996, graduating with First class honors (Cum Laude). She later joined a middle level college and pursued an International Diploma in Computer Science from 1999 to 2000. In 2005 she joined Kenyatta University for Msc Applied Mathematics and graduated in the year 2007 with a project in Computational Fluid Dynamics. From 2011 to 2014, Winifred was at Cape Peninsula University of Technology in South Africa studying for her Doctorate in Philosophy in Applied Mathematics. She graduated in the year 2014 with a Thesis titled “Analysis of Hydro magnetic Boundary layer flow and Heat Transfer of Nanofluids. She has widely published in International Journals, has participated and presented papers in several International conferences and Workshops. She has supervised and still is supervising students for Msc and PhD. She has previously held position as a Chairperson for African Women in Mathematics Association (AMWA-Kenya) and is currently the Vice-Secretary for the same Association. Currently she works as a Lecturer in the Department of Pure and Applied Mathematics at Kenyatta University. She also works as a Volunteer tutor and mentor.