International Journal of Applied Science and Mathematics
Volume 4, Issue 1, ISSN (Online): 2394-2894

Differential Transform Method for a Class of Nonlinear Integro-Differential Equations with Rational Derivative-Type Kernel

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Date of publication (dd/mm/yyyy): 17/02/2017

Abstract – **This paper implements the differential transform method for a class of integro - differential equations.**
The numerical results obtained agree with other numerical methods solutions and is very simple to realize and one can obtain solution of arbitrary order of accuracy.

Keywords – **Integro-Differential Equation, Differential Transform Method, Numerical Simulation.**

I. Introduction

We consider the nonlinear differential equation in the form

$u''(t) = P(t,u,u') + Q(t,u,u') \int_0^t R(t,u,u'(s))\, ds$

where P and Q are polynomials in u and u' and R is a rational function with u' only at the numerator.

Our aim is to prove that the Differential Transform Method ***(DTM)*** can be applied successfully.

The DTM is simple to implement compared to numerical method found in the literature. It is based on simple mathematical tools.

We outline in this paper the use of DTM in solving the class of integro-differential equation arising in the modeling of some physical phenomena in mechanics, chemistry, biology, epidemiology and others.

We recall the following elementary properties used in defining the method and doing calculations.

II. Structure of the Differential Transform Method

Following the work in [1] and [2] the initial function u (t) is supposed analytic in the domain D.

We defined the differential transform at point 1_0 to be U (k) or some time just denoted U_k by :

$$\boldsymbol{U} = \frac{1}{k!}\left[\frac{d^k u(t)}{dt^k}\right]_{t=to} \qquad (8)$$

The following properties can be easily computed from the definition

$$P1: w(t) = u(t).v(t)$$

$$\boldsymbol{W(k)} = \sum_{k=0}^{k} U_l V_{k-l}$$

$$\boldsymbol{P2: w(t) = au(t) + \beta v(t)}$$

$$W(k) = \square \boldsymbol{U_k} + \square \boldsymbol{V_k}$$

$$P3: w(t) = \int_0^t v1(s)v2(s)ds$$

$$w(k) = \frac{1}{k}\sum_{k=0}^{k-1} V_u V_{2k-l-1}$$

$$P4: w(t) = u^m(t)$$

$$W(k) = \sum_{l=0}^{k} U(l)U^{m-1}(k-1) \text{ this can be } iterated$$

U^p *being the notation for the DTM of* $U^p(t)$

The inverse formula of the ***(DTM)*** for W (k) is given by

$u(t) = \sum_{k=0}^{\infty} U_k t^k$, (9)

Here we have taken $t_0 = 0$.

For the details see [6]; [7] *and* [8]

III. Applications

Let us consider the following examples that are going to be solved by the *(DTM)*

Example 1

Let us consider the following equations:

$$u''(t) - 0.5u'(t)u(t) + u(t) + \int_0^t \frac{u^2(s)}{(1+u^2)}ds = f_1(t)$$

$u(0) = -1;\ u'(0) = 1$

Example 2

$$u''(t) - u^3(t) + 2u'(t)\int_0^t \frac{u(s)u^{3'}(s)}{(1+u^2)}ds = f_2(t)$$

$u(0) = 1;\ u'(0) = 1$

Example 3

$$u''(t) - 0.5u'(t) + u(t) \int_0^t \frac{u'u'^2(s)}{\sqrt{(1+u^2)}}ds = f_3(t)$$

Example 4

$$u''(t) - u'(t) + 2u(t) \int_0^t \frac{u'^2(s)}{\sqrt{(1+u^2)}}ds = f_4(t)$$

$u(0) = 1;\ u'(0) = 1$

The second member are defined by

$$f_1(t) = \frac{-1}{2}cos2t + \int_0^t \frac{1 - sin2s}{2sin2s} ds$$

$$f_1(t) = -2e^{-t} + 2e^{-t}\left(\frac{1}{5}e^{5(-t)} - \frac{1}{3}e^{3(-t)} + e^{-t} + \frac{1}{4}\pi + \arctan(e^t) - \frac{13}{15}\right)$$

$$f_3(t) = \frac{1}{2}e^t + e^t \int_0^t \frac{e^{3s}}{\sqrt{1 + e^{2s}}} ds$$

$$f_3(t) = \frac{1}{2}e^t + e^t\left(\frac{1}{2}ln\left(e^t + \sqrt{e^{2t} + 1}\right) - \frac{1}{2}ln(\sqrt{2} + 1) - \frac{1}{2}e^t\sqrt{e^{2t} + 1} + \frac{1}{2}\sqrt{2}\right)$$

$$f_4(t) = e^t + 2te^t \int_0^t \frac{s^2e^{2s}}{\sqrt{1 + s^2e^{2s}}} ds = e^t + 2te^t \int_0^t s^2\frac{e^{2s}}{\sqrt{s^2e^{2s} + 1}} ds$$

The exact solution for example 1 is $u_1(t) = cost - \sin t$
For the second example is $u_2(t)=e^{-t}$
For example 3
$u_3(t) = e^t$
For example 4
$u_4(t) = te^t$

The solutions are constructed for large values of t and for small values of t.

The kernel of the examples given is in the form
K (u, *u'*) = k_1 *(u)* k_2*(u')*

The integral part of the above differential operator can be computed using the formula (8) *and* the properties Pi, i=1, 2, 3, 4.

[4], [5], [6] and [7] give more details.

IV. Numerical Results

We take different Kernel *(rational)* to obtain the same exact solutions we obtain very similar results of that of Borhanifar [3] given in the table below. In a forthcoming paper we are going to give more details on these calculations and a complete matlab program for evaluating these results.

Table 1

t|*error*→	n=5	n=10	n=15	n=20
0.2	0.8628(-7)	0.5551 (-15)	0.0	0.0
0.4	.5348(-5)	. 1084(-11)	0.0	0.0
0.6	0.5886(-4)	0.9522(-10)	0.5551 (-16)	0.5551 (-11)
0.8	0.1387(-3)	0.2286(-8)	0.1314(-14)	0.3122 (-16)
1	01168(-2)	0.2696(-7)	0.44.90(-13)	0.1110 (-15)

Numerical for example 1

t↓*error*	n=5	n=10	n=15	n=20
0.2	0.8641 (-7)	0.5551 (-15)	0.0	0.0
0.4	.5379(-5)	.1016(-11)	0.2220(-15)	0.2220(-15)
0.6	0.5963(-4)	0.8654(-10)	0.1110(-15)	0.1110(-15)
0.8	0.1213(-2)	0.2311 (-7)	0.1276 (-14)	0.1110(-15)
1	0.3263(-3)	0.2016(-8)	0.4501(-13)	0.0

Numerical results for example 2

Table 3

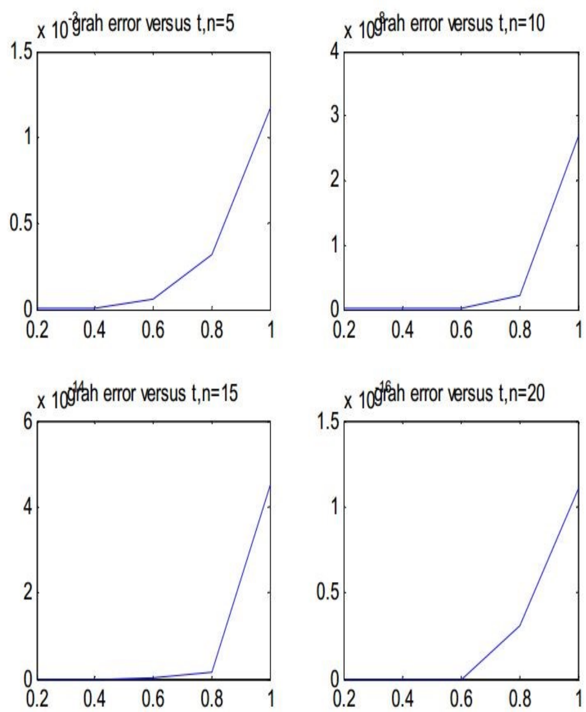
t↓*error*	n=5	n=10	n=15	n=20
0.2	0.9149(-7)	0.6661 (-15)	0.2220 (-15)	0.2220(-15)
0.4	0.6030(-5)	. 1087 (-11)	0.2220(-15)	0.2220(-15)
0.6	0.7080(-4)	0.9576(-10)	0.2220(-14)	0.2220(-15)
0.8	0.4102(-3)	0.2304(-8)	0.1332(-14)	0.0
1	01615(-2)	0.2731 (-7)	0.5062(-13)	0.0

Numerical results for example 3

Table 4

t↓*error*	n=5	n=10	n=15	n=20
0.2	0.8629(-7)	0.1387(-15)	0.5551 (-16)	0.5551 (-16)
0.4	0.2412(-5)	0.4348(-12)	0.1110(-15)	0.1110(-15)
0.6	0.4248(-4)	0.5739(-10)	0.2220(-15)	0.2220(-15)
0.8	0.3282(-3)	0.1843(-8)	0.1110(-14)	0.0
1	01615(-2)	0.2731 (-7)	0.5062(-13)	0.0

Numerical results for example 4



V. Conclusion

The aim of this paper was to confirm through some examples make of rational function in u and u' the validity of the Differential Transform method for such integro-differential equations.

The aim has been achieved through the numerical results obtained.

Acknowledgements

This work was carried out under the financial support of the University of Calabar trough the visiting status grant offered to the first author and the ministry of high education of Cameroon through the special grant. For research . The authors would like to thank the reviewers of this work.

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