

# Application of Case Teaching in the Course of Linear Algebra

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**Abstract** – This paper analyzes the significance of case teaching method for linear algebra teaching, introduces the application of case teaching in inverse matrix and matrix diagonalization, and improves the quality of linear algebra teaching by introducing practical application problems into linear algebra teaching. It provides reference advice to teachers.

**Keywords** – Case Teaching, Typical Case, Linear Algebra Teaching, Inverse Matrix, Diagonalization of Matrices

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## I. INTRODUCTION

### 1.1 Introduction to Linear Algebra Course

Linear algebra, like calculus, is one of the two major introductory courses in higher mathematics. It is a very good mathematics course and a very good tool course. It tells the basic knowledge and theory of matrix theory and linear equations. The main content also includes determinants, vector spaces, eigenvalues, similar matrices, and positive definite matrices [1].

Linear algebra has important applications in physics and many technical disciplines, so it occupies a leading position in various algebraic branches. Today, when computers are widely used, computer graphics, computer-aided design, cryptography, virtual reality and other technologies all use linear algebra as part of their theory and algorithm. Linear algebra embodies the connection between geometric concepts and algebraic methods. The axiomatic method abstracted from specific concepts is very useful for strengthening people's mathematical training. With the development of science, it is not only necessary to study the connection between individual variables, but also to further study the connection between multiple variables. Various practical problems can be linearized in most cases. With the development of computers, the linearization problem can be calculated again [2]. Linear algebra is a powerful tool for solving these problems.

### 1.2 Introduction to Case Studies

Case teaching method is a case-based teaching method. The case essentially proposes an educational dilemma. There is no specific solution, and teachers play the role of designers and motivators in teaching. Encouraging students to actively participate in the discussion, unlike traditional teaching methods, the teacher is a very knowledgeable person and plays the role of a knowledge giver. Case teaching originated at Harvard Business School in the United States. At that time, teaching took a very unique form of case study. These cases were all from the real situation or event of business management. Through this method, they helped to cultivate and develop students' active participation in classroom discussions. After implementation, it is quite successful [3].

Case teaching allows students to establish a set of complete and rigorous logical thinking methods and

thinking methods suitable for themselves in the reading, thinking, analysis, and discussion of cases through one or more unique and representative typical events. In order to improve students' ability to analyze problems and solve problems, they will improve their quality.

### 1.3 The Significance of Case Teaching to Linear Algebra Curriculum

In the course teaching of linear algebra, students' understanding of linear algebra knowledge is realized by constructing the case teaching situation and strengthening the students' practice training. As far as students are concerned, it is difficult to understand linear algebra knowledge. Teachers help students visualize linear algebra knowledge is the first problem. Through case teaching, teachers bring the relationship between classroom teaching and practice life closer, integrate various teaching resources, and make linear algebra knowledge three-dimensional. The enhanced communication between teachers and students through the case can have a spiritual resonance, allowing students to experience the practical effectiveness of linear algebra.

## II. APPLICATION OF CASE TEACHING MODEL IN LINEAR ALGEBRA COURSE

In linear algebra teaching, teachers reasonably create problem situations, follow the principle of students' interest in learning, strengthen teaching guidance, rationally set up case situations, effectively highlight the role of case teaching methods, and improve the quality and teaching efficiency of linear algebra classroom teaching [4]. The following introduces the application of case teaching in two important knowledge points of linear algebra.

### 2.1 Application of Case Teaching in Inverse Matrix Teaching

Matrix operations occupy a large proportion of linear algebra teaching, especially the inverse of matrices. Conceptual abstraction is not easy to understand, and computation is not easy to grasp. To achieve the improvement of understanding speed and problem solving efficiency, it is necessary to apply case teaching model rationally. Construct the knowledge structure of image to help carry out teaching work.

#### 2.1.1 Case Introduction

When sending a message, it is usually encrypted for the sake of confidentiality. When the recipient receives the encrypted message, it needs to decrypt it to understand the true meaning of the message. This is cryptography. In cryptography, there is an encryption method that matches 26 Arabic numerals and 26 English letters one by one. When sending a message, you must first convert the message into a number. For example, when sending a message with the content "EFG" (we call it explicit), we first convert it to the numbers 5, 6, and 7, and we use a matrix  $X = (5, 6, 7)^T$  to represent it. Then encrypt it. The encryption method is multiplied by the matrix  $A$  on the left side of  $X$ , where the matrix  $A$  is called the encryption matrix.

Such as

$$A = \begin{pmatrix} -1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{pmatrix},$$

let  $Y = AX = (2, 13, 12)^T$ , and it's an encrypted message. The corresponding letter is "BML". When the recipient receives the encrypted message, how to decrypt it?

#### 2.1.2 Case Analysis and Answers

Through the introduction of the problem of how to crack encrypted messages, it attracted the attention of students and stimulated their interest in learning. At this point, the teacher can logically introduce the definition and calculation of the following inverse matrix [5].

After teaching the definition and calculation of the inverse Matrix, the topic is transferred to the question of how to crack the encrypted message at the beginning. Originally, as long as the encrypted matrix  $A$  used by the message in encryption is an invertible Matrix, the inverse matrix of  $A$  is obtained, and then use this inverse matrix to multiply left by both ends of the equation  $Y = AX$ , we have  $X = A^{-1}Y = (5, 6, 7)^T$ , so that the decryption of the message is completed. Therefore, the two parties only need to agree on the encryption matrix in advance. When the receiver receives the encrypted message, the inverse matrix can be used to decrypt it. This not only answers the questions raised at the beginning, but also allows students to experience the charm of mathematics: mathematics originates from practice and is applied to practice.

## 2.2 Application of Case Teaching in Matrix Diagonalization Teaching

### 2.2.1 Case Introduction

In two different cities, E and F, the number of people moving into F City from E City is 30 % each year, and the number of people moving to E City from F City is 20 % each year. Assume that the population of City E is 6 million and the population of City F is 4 million. If the total number of people in the two places does not change, what are the population numbers of City E and City F three years later? After a few years, what will the population of the two cities tend to be?

### 2.2.2 Case Analysis

First, we need to calculate the population of City E and City F after the first year, set the matrix  $A = \begin{pmatrix} 0.7 & 0.2 \\ 0.3 & 0.8 \end{pmatrix}$ .

The initial population of cities E and F is expressed as a matrix  $x_0 = \begin{pmatrix} 600 \\ 400 \end{pmatrix}$ . The matrix multiplication can be used to calculate

$$x_1 = Ax_0 = \begin{pmatrix} 500 \\ 500 \end{pmatrix}.$$

Analogous analogy,

$$x_2 = Ax_1 = A^2x_0, \\ x_3 = Ax_2 = A^3x_0, \dots, x_n = A^n x_0.$$

To calculate  $A^3$  and  $A^n$ ,  $A$  needs to be diagonalized. At this point, teachers can teach the relevant definitions, theorems, and steps of diagonalization of matrices [6].

### 2.2.3 Case Answers

After learning the relevant knowledge of diagonalization, we can calculate that the eigenvalue of  $A$  is  $1, \frac{1}{2}$ . So we get the diagonal matrix

$$\Lambda = \begin{pmatrix} 1 & 0 \\ 0 & \frac{1}{2} \end{pmatrix}.$$

The corresponding feature vectors are  $(2, 3)^T$ ,  $(1, -1)^T$ , the similarity transformation matrix is

$$P = \begin{pmatrix} 2 & 1 \\ 3 & -1 \end{pmatrix}.$$

So

$$A = P \Lambda P^{-1},$$

$$A^3 = P \Lambda^3 P^{-1} = \begin{pmatrix} \frac{19}{40} & \frac{14}{40} \\ \frac{21}{40} & \frac{26}{40} \end{pmatrix},$$

$$A^n = P \Lambda^n P^{-1} = \begin{pmatrix} \frac{2}{5} + \frac{3}{5 \cdot 2^n} & \frac{2}{5} + \frac{-2}{5 \cdot 2^n} \\ \frac{3}{5} + \frac{-3}{5 \cdot 2^n} & \frac{3}{5} + \frac{2}{5 \cdot 2^n} \end{pmatrix}.$$

Hence

$$x_3 = A^3 x_0 = \begin{pmatrix} 4.25 \\ 5.75 \end{pmatrix}.$$

That is, after three years, the population of E City will be 4.25 million, and the population of F City will be 5.75 million.

Since

$$\lim_{n \rightarrow \infty} x_n = \lim_{n \rightarrow \infty} A^n x_0 = \begin{pmatrix} 4 \\ 6 \end{pmatrix},$$

therefore, after a few years, the population of City E tends to 4 million, and the population of City F tends to 6 million.

### III. CONCLUSION

Combining the application of the above case teaching methods in inverse matrix and matrix diagonalization teaching, starting from practical problems, the abstract knowledge is visualized, and the students have mastered the abstract knowledge of linear algebra while thinking about case problems. The understanding of knowledge has become more thorough [7].

In the application of case teaching methods, teachers should also do a good job in streamlining cases and strengthen the fit between cases and classroom knowledge points [8]. Usually each knowledge point can introduce one or two cases. Streamlined cases allow students to understand the core content, help students remember definitions and theorems, and will find ways to apply the linear algebra knowledge they have learned to solve problems.

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