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The Fractional Base Power Identity of A Logarithm

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Abstract – **Logarithm has long been a tool for simplifying calculations to a much more user-friendly form. Common logarithm of a given number *x* is the exponent to which another fixed number, the base *b*, must be raised to produce that number *x*. The purpose of this study is to focus on logarithm with the base $\frac{a}{b}$ where $a, b \in Z^{+}$. The study introduced a new fractional base power identity of logarithm and demonstrate its application on road and the mining industry. This is of interest because the new fractional base identity can be used in other fields such as, Richter Scale; for measuring the magnitude of an earth quake, pH balance; for measuring the acidity and alkalinity of a substance.**

Keywords – **Logarithm, Common Logarithm, Richter Scale, Ph Balance, Fractional Base Power Identity.**

I. Introduction

John Napier was the first mathematician to devise the idea of the logarithm, naming it logarithm from the Greek *roots logos*, meaning proportion, and *arithmus*, meaning number, because he used it to relate numbers to another value [4]. Napier developed the logarithm for the use in the field of astronomy (Clark p. 4) as the calculation that were being done in this field were complex and required the multiplication of very large numbers. Henry Briggs introduced common (base 10) logarithms, which were easier to use [2] [3]. Tables of logarithms were published in many forms over four centuries. The idea of logarithms was also used to construct the slide rule, which became ubiquitous in science and engineering until the 1970s. A breakthrough generating the natural logarithm was the result of a search for an expression of area against a rectangular hyperbola, and required the assimilation of a new function into standard mathematics [4] [2] [3] [8]. In this regard, logarithms are a way of showing how big a number is in terms of how many times you have to multiply a certain number (the base) to get it. For example, $\log_2 512 = 9$, as $512 = 2^9$ and also $\log_5 25 = 2.$, as $5^2 = 25$, etc.

Fractional base power identity is used to solve logarithm with the base being a fraction. For example, $\log_{\frac{1}{n}} \frac{B}{A}$, where $n \geq 1$ and $A, B \in Z^{+}$.

II. Preliminaries

2.1. *Logarithm*

2.1.1. *Definition*

If $a > 0$ and b is a constant $(b \neq 0)$ then $y = log_b x$ if and only if $b^y = x$.

2.1.2. *Logarithms of Powers* $log_a x^n = n log_a x$

Proof

Let $b = log_a x$

$$a^b = x \quad (1)$$

Mult. through the exponents of eqn. (1) by n,

$(a^b)^n = x^n$

$a^{bn} = x^n$

Take log to base a on both sides,

$log_a\, a^{bn} = log_a\, x^n$

$bn = log_a\, x^n$

$\therefore log_a\, x^n = n\, log_a\, x$

2.2. *Change of Base of Logarithm*

2.2.1. *Change of Base Law*

Tables and calculators can only be used to compute logarithms, which are of base 10. Change of base enables us to compute logarithms which are of different base other than 10.

$$log_a\, x = \frac{log_b\, x}{log_b\, a}$$

Proof

Let $y = log_a\, x$, $x = a^y$

Take log of base b on both sides,

$log_b x = log_b a^y$

$y\, log_b a = log_b x$

$y = \frac{log_b x}{log_b a}$

$log_a x = \frac{log_b x}{log_b a}$

2.3. *Concept of the Fractional Base Power Identity*

(1) If the number to the log is a fraction with a fractional base, where the numerator of the number (not base) is less than the denominator then, the results obtained will be a positive.

(2) If the number to the log is a fraction with a fractional base, where the numerator of the number (not base) is more than the denominator then, the results obtained will be negative.

(3) If the number to the log is a whole number with a fractional base, also the results obtained will be a negative.

III. Main Result

3.1. *Fractional Base Power Identity Law*

For x, its fractional base inverse $= \frac{1}{x}$, For $\frac{1}{-x}$, its fractional base inverse $= -x$.

For $-x$, its fractional base inverse $= -\frac{1}{x}$, For $\frac{1}{x}$, its fractional base inverse $= x$.

3.1. *Theorem (Fractional Base Power Inverse)*

Given that $\log_{\frac{1}{a}}(B)$, for all value of "a" belonging to a set of natural numbers, we then transform the "$\frac{1}{a}$" into "a" by the laws to obtain a^{-1}. We then take the -1 and transform it into power identity form, as $-\frac{1}{1}$ then multiple it by each base power and the number power as $\log_{a^{-1\times\frac{1}{-1}}}\left(b^{1\times-\frac{1}{1}}\right)$ to obtain $\log_a\left(B^{\frac{-1}{1}}\right)$.

3.1.1. *Examples*

Evaluating $log_{\frac{2}{4}} 4$ we can write this as $log_{\frac{1}{2}} 4 = \log_{2^{-1}} 2^2$

Now multiplying the power by the base power inverse, that is $-1 = \frac{1}{-1} = \log_{2^{-1\times\frac{1}{-1}}}\left(2^{2\times\frac{1}{-1}}\right) = \log_2 2^{-2}$

$= -2\log_2 2$ but $\log_2 2 = 1$

$\Rightarrow -2 \times 1 = -2$

$\therefore\ log_{\frac{2}{4}} 4 = -2$

3.1.2**.** *Example*

To find the value of x in $x\log_{\frac{1}{7}} 49 = x^2 + \log_{\frac{1}{2}} 16$, we rewrite the equation as $x(\log_{7^{-1}} 7^2) = x^2 + (\log_{2^{-1}} 2^4)$

$x(\log_7 7^{-2}) = x^2 + (\log_2 2^{-4})$

$x(-2\log_7 7) = x^2 + (-4)\log_2 2$

$x(-2) = x^2 + (-4)$

$-2x = x^2 - 4$

$x^2 + 2x - 4 = 0$

Solving simultaneously result $x = 1.24$, $x = -3.24$

3.3. *Fractional Change of Base*

Given that $\log_{\frac{A}{B}} D$, where A is either odd number or prime number. Let a be the base number.First, finding the base, we need to convert fractional base, $\frac{A}{B}$ into a whole number base say a. Thus let $a = A \times \frac{1}{B} \times \frac{1}{A} \Rightarrow a = \frac{1}{B} \therefore \log_{\frac{A}{B}} D = \frac{\log_{\frac{1}{B}} D}{\log_{\frac{1}{B}}\frac{A}{B}}$.

3.3.1. *Example*

Evaluating $log_{\frac{2}{7}} 4$, let a be the base number.

First, finding the base, we need to convert fractional base, $\frac{A}{B}$ into a whole number base say a.

Let $a = 2 \times \frac{1}{7} \times \frac{1}{2} \Rightarrow a = \frac{1}{7}$

$$\Rightarrow \log_{\frac{2}{7}} 4 = \frac{\log_{\frac{1}{7}} 4}{\log_{\frac{1}{7}} \frac{2}{7}} = \frac{0.60205}{0.301029 - 0.845098} = -1.1065$$

$$\therefore \log_{\frac{2}{7}} 4 = -1.1065$$

3.4. *Practical Application on Fractional Base Power Identity*

This aspect focuses on the application of the fractional base power identity on road assessment and the impact on the blasting site in mining industry.

Let's consider a given road with distances from a given point, say *A* and *B* on which a person crosses to determine its closest point of impact [1] [5] [7].

Fig. 1. Road assessment between *A* and *B*.

3.4.1. *Checking the Closest Direction of Impact*

Let *A* be the distance (12,884m) from the person to car *A* and let *B* be the distance (15,672m) from the person to car *B*,

From the concepts

If $\log_{\frac{1}{n}} \left[\frac{B}{A}\right]$, where *B* is greater than *A* the results will be negative.

If $\log_{\frac{1}{n}} \left[\frac{A}{B}\right]$, where *A* is greater than *B* the results will be positive.

- $\log_{\frac{1}{10}} \left[\frac{15{,}672}{12{,}884}\right] = -0.0851$
- $\log_{\frac{1}{10}} \left[\frac{12{,}884}{15{,}672}\right] = 0.0851$

Reason for the negative result shows an impact on the A to be more.

Reason for the positive result shows an impact on the B to be less.

3.4.2. *Deduction*

The less negative result is stronger than the more negative result. This means that there would be more impact on A as compare to B.

Mining Industry 3.4.3: Again, let's consider amining site and then assume a population of 30,000 people away from each side A and B of the mining site. A is a town of a distance of 11,000 meters away from the mining site and B is 11,600 meters away from the mining site [1] [5] [7]. This is shown below:

Fig. 2. Blasting Area.

Now we then apply the concepts of the fractional base power identity to predict the direction of impact.

Let 11,000 meters be the distance from a town *A* to the mining site.

Let 11,600 meters be the distance from a town *B* to the mining site.

Again, from the concepts,

If $\log_{\frac{1}{n}}\left[\frac{B}{A}\right]$, where B is greater than A the results will be negative.

If $\log_{\frac{1}{n}}\left[\frac{A}{B}\right]$, where A is greater than B the results will be positive.

- $\log_{\frac{1}{10}}\left[\frac{11{,}600}{11{,}000}\right] = -0.0231$
- $\log_{\frac{1}{10}}\left[\frac{11{,}000}{11{,}600}\right] = 0.0231$

Reason for the negative result shows an impact on the *A* to be more.

Reason for the positive result shows an impact on the *B* to be less.

3.4.4. *Deduction*

This implies that the population at the distance of 11,000*m* must move further from the mining site to escape any misfortunes.

IV. Conclusion

In this study, we tried to focus on the fractional base of logarithm by introducing new technique (called fractional base identity) to solve a complex logarithm with the base being a fraction from the common logarithm and used it to predict real life problems.

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