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MHD Stagnation-Point Flow over a Stretching Sheet in the Presence of Thermal Radiation and Chemical Reaction in a Porous Medium

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Abstract **– This paper deals with a steady two-dimensional magnetohydrodynamic stagnation-point flow, heat and mass transfer of a viscous fluid over a stretching sheet in the presence of thermal radiation, heat generation/ absorption and chemical reaction of order n in a Darcian porous medium. The basic boundary layer equations for flow, heat and mass transfer describing the problem are nonlinear partial differential equations. These equations are transformed into coupled nonlinear ordinary differential equations by means of similarity transformation and are then solved numerically using a suitable finite difference technique. The finite difference schemes are employed to linearize the governing ordinary differential equations and the resulting nonlinear algebraic equations are solved using modified Newton's method. The influences of different embedded flow parameters such as permeability, magnetic field, stretching ratio, rate of chemical reaction, order of chemical reaction and thermal radiation parameters as well as Prandtl and Schmidt numbers on velocity, temperature and species concentration profiles are presented in graphical forms. The local skin-friction, Nusselt and Sherwood numbers are tabulated and discussed in details.**

Keywords **– Magnetohydrodynamics, Stretching Sheet, Thermal Radiation, Chemical Reaction.**

I. INTRODUCTION

Boundary layer fluid flow problems in different dimensions over stretching sheet with heat and/or mass transfer effects have numerous engineering and industrial applications. They include glass blowing melt spinning, heat exchanger design, fiber and wire coating, production of glass fibers, industrialization of rubber and plastic sheets, etc. Sakiadis [1] was the pioneer in formulating the problem of a steady two-dimensional boundary layer flow due to a stretching sheet. Many investigators such as [Gupta and Gupta [2], Vajravelu and Nayfeh [3], Datta *et al*. [4], Siddheshwar and Mahabaleshwar [5], Bhattacharyya [6], Yasmin et al. (7), Baoku et al. [8], etc] have extended the work of Sakiadis [1] in order to gain more understanding for flows of different fluids over linear or non-linear stretching sheets with heat and/or mass transfer phenomena.

The study of magnetohydrodynamics has considerable interest in the technical fields due to its important applications in industrial technology. These applications comprise MHD power generators, cooling of nuclear reactions, liquid metal flow control, biological transportation, high temperature plasmas, micro MHD pumps, drying processes, solidification of binary alloy, etc. The mass transfer of the steady two-dimensional magnetohydrodynamic boundary layer flow of an upper-convected Maxwell fluid past a porous shrinking sheet in the presence of chemical reaction was investigated by Hayat et al. [9]. They used homotopy analysis method (HAM) to obtain series solutions for the problem. Khater et al. [10] presented a weakly nonlinear theory of wave propagation in superposed fluids in the presence of magnetic field. They examined the nonlinear evolution of Rayleigh-Taylor instability in three dimensions in the context of magnetohydrodynamics and performed a stability analysis of their solutions. Baoku et al. [11] studied the influence of magnetic field, third grade, partial slip and other thermophysical parameters on the steady flow, heat and mass transfer of viscoelastic third grade fluid past an infinite vertical insulated plate subject to suction across the boundary layer. It was concluded that the magnetic field strength was found to decrease with an increasing temperature distribution when the porous plate was insulated.

Recently, the entropy generation in a magnetohydrodynamic flow and heat transfer of a Maxwell fluid was reported by Shateyi et al. [112]. They obtained the solutions to the problem using the spectral relaxation method. The magnetohydrodynamic flow of a generalized Maxwell fluid induced by a moving plate, where the second-order slip between the wall and the fluid was examined by Liu and Guo [13]. They pointed out that the velocity corresponding to flows with slip condition subjected to magnetic field is lower than that with first-order slip condition. Gaffar et al. [14] analyzed the nonlinear, isothermal, steady-state, free convection boundary layer flow of an incompressible third grade viscoelastic fluid past an isothermal inverted cone in the presence of magnetohydrodynamic, thermal radiation and heat generation/absorption. They employed the second-order accurate implicit finite-difference Keller-Box Method to obtain solutions for the transformed conservation equations for linear momentum, heat and mass subject to the realistic boundary conditions. Olajuwon et al. [15] considered the hydromagnetic partial slip flow, heat and mass transfer of a third grade non-Newtonian fluid over a vertical surface in the presence thermal radiation in an optically thick environment through a porous medium. They employed the midpoint integration scheme alongside Richardson’s extrapolation technique to obtain numerical solutions to the problem.

Flows in porous media have gained the attention of many researchers because of their applications in astrophysics, geothermal and oil reservoir engineering. Flows involving viscous fluids have been studied extensively. One can make references to the works Vafai and Tien [16], Raptis [17], Ingham and Pop [18], Vafai [19], Pop and Ingham [20], Ingham et al. [21], Ingham and Pop [22], Vafai [23], Nield and Bejan [24],] and references therein concerning viscous flow through Darcian and non-Darcian porous media with different geometries for the flows. Some researchers have also focused on convective heat transfer with thermal radiation through porous media. Many recent engineering processes occur at high temperatures. Therefore, the knowledge of radiation heat transfer besides the convective heat transfer plays very crucial roles and cannot be neglected. Gas turbines, nuclear power plants and several propulsion devices for missiles, satellites aircraft and space vehicles are few examples of such engineering areas involving thermal radiation. (see Eckert and Drake [25]). Many researches on influences of thermal radiation on viscous fluids over different surfaces have been conducted by Gorla [26], Raptis and Massalas [27], Pop et al. [28], Abdulhakeem and Sathiyanathan [29], Hossain et al. [30], Hayat et al. [31] and Baoku et al. [32].

The present study focuses on the two dimensional, incompressible and electrically conducting fluid near a stagnation point on a non-conducting porous permeable stretching sheet in the presence of uniform heat generation/absorption, radiative heat transfer and chemical reaction of order n in a Darcian porous medium. Similarity transformation has been employed to non-dimensionalize the governing equations and numerical finite difference technique with modified Newton’s method is used to obtain solutions to the problem. These are presented in graphical forms for various values of the pertinent parameters. To gain thorough insight into the physics of the problem such as the surface shear stress, rates of heat and mass transfer, numerical results for local skin friction coefficient, Nusselt and Sherwood numbers for some values of the embedded parameters are displayed in a table. Thus, the present investigation becomes the basis of some engineering applications.

II. Governing Equations

A steady, two-dimensional flow of an incompressible magnetohydrodynamic fluid near the stagnation point on a permeable stretching sheet in the presence of thermal radiation, heat generation/absorption and chemical reaction of order n in a Darcian porous medium is examined. The fluid is occupied in the region $y>0$ driven by a stretching sheet located at $y=0$ with a fixed stagnation point at $x=0$. Choosing the x and y axes along and perpendicular to the sheet, the stretching velocity and the free stream velocity are assumed to vary proportional to the distance x from the stagnation point, i.e. $U_w(x)=ax$ and $U_w(x)=bx$ where a and b are rate constants. It is also assumed that a uniform magnetic field of strength B_0 is applied in the positive y - direction normal to the sheet. The induced magnetic field due to motion of the electrically conducting fluid is neglected. The temperature and species concentration at the sheet are maintained at the prescribed constant values T_w and C_w respectively. The ambient values attained at the free stream of T and C are T_∞ and C_∞ respectively. Assuming all the thermo-physical properties are constant, the two-dimensional boundary layer equations, governing the flow of a steady, laminar and incompressible viscous fluid are:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=U\frac{dU}{dx}+\upsilon\frac{\partial^2 u}{\partial y^2}-\frac{\sigma\beta_0^2}{\rho}\left(U-u\right)-\frac{\upsilon\phi}{K'}u \tag{2}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\frac{\partial^2 T}{\partial y^2}+\frac{1}{\rho Cp}Q\left(T-T_\infty\right)-\frac{1}{\rho Cp}\frac{\partial q_r}{\partial y} \tag{3}$$

$$u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}=D_m\frac{\partial^2 C}{\partial y^2}-k_0\left(C-C_\infty\right)^n \tag{4}$$

where x, y, u, v, T, C, υ, σ, B_0, ρ, C_p, α, Q, f, k_0, q_r, D_m, K' and n are coordinate axes along the stretching sheet in the direction of motion and normal to it, velocity components in the directions of x and y axes, fluid temperature inside the boundary layer, species concentration of the fluid, kinematic viscosity, electrical conductivity, magnetic field flux, fluid density, specific heat at constant pressure, thermal diffusivity, internal heat generation/absorption, porosity, permeability of the medium, radiative heat flux, mass diffusivity, rate of chemical reaction and order of chemical reaction respectively.

The boundary conditions for the problem are:

$$u=U_w\left(x\right), v=-V_w, T=T_w, C=C_w \text{ at } y=0 \tag{5}$$

$$u\to U_\infty=0,\ T\to T_\infty, C\to C_\infty \text{ as } y\to\infty \tag{6}$$

Following Cogley et al. [33], it is assumed that the fluid is optically thin with a relatively low density and the radiative heat flux is given by:

$$\frac{\partial q_r}{\partial y} = 4\xi^2 (T - T_\infty) \tag{7}$$

where ξ is the mean radiation absorption coefficient.

Dimensionless quantities are introduced to simplify the mathematical analysis of the problem by introducing the following similarity transformation:

$$\eta = \sqrt{\frac{c}{\upsilon}} y, \psi = \sqrt{c\upsilon}\, x f(\eta), \theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty}, \ \phi(\eta) = \frac{C - C_\infty}{C_w - C_\infty} \tag{8}$$

The continuity equation (1) is automatically satisfied by chosen a stream function $\psi = (x, y)$ as:

$$u = \frac{\partial \psi}{\partial y}, \ v = -\frac{\partial \psi}{\partial x} \tag{9}$$

Using the above similarity transformation and equations (5) – (7), the governing equations (2)-(4) are transformed to the following coupled nonlinear ordinary differential equations:

$$f'''(\eta) + f(\eta) f''(\eta) - f'^2(\eta) + \varepsilon^2 - M(\varepsilon + f'(\eta)) - P f'(\eta) = 0 \tag{10}$$

$$\theta''(\eta) + \Pr f(\eta)\theta'(\eta) - \Pr(Rt - H)\theta(\eta) = 0 \tag{11}$$

$$\phi''(\eta) + Sc\, f(\eta)\phi'(\eta) - \gamma\phi^n(\eta) = 0 \tag{12}$$

with boundary conditions:

$$f(\eta) = S, \ f'(\eta) = 1, \ \theta(\eta) = 1, \ \phi(\eta) = 1 \text{ at } \eta = 0 \tag{13}$$

$$f'(\eta) \to \varepsilon, \ \theta(\eta) \to 0, \ \phi(\eta) \to 0 \text{ as } \eta \to \infty \tag{14}$$

where the parameters are defined as:

$S = \frac{-V_w}{\sqrt{c\upsilon}}$ is the is the suction velocity parameter, η is the similarity variable, prime is the differentiation with respect to η, f', θ and ϕ are the dimensionless velocity, temperature and species concentration respectively, $M = \frac{\sigma\beta_0^2}{cx^2\rho}$ is the Hartmann number, $\Pr = \frac{\upsilon}{\alpha}$ is the Prandtl number, $P = \frac{\upsilon\varphi}{cx^2K'}$ is the permeability parameter, $Sc = \frac{\upsilon}{D_m}$ is the Schmidt, $\varepsilon = \frac{a}{b}$ is the stretching rate, $\gamma = \frac{k_0}{c}(C_w - C_\infty)^{n-1}$ is the rate of chemical reaction, $H = \frac{Q}{c\rho Cp}$ is the heat source/sink parameter and $R = \frac{4\xi^2 a}{c\rho Cp}$ is the thermal radiation parameter.

III. Method of Solution

The steady coupled nonlinear ordinary deferential equations (10) - (12) with the boundary conditions (13) and (14) are solved by employing the centred finite difference scheme. The finite difference method (FDM) approximates the derivatives by using the appropriate difference equations to discretize the nonlinear governing differential equations. The idea is to replace the derivatives appearing in the differential equations by difference quotients that approximates differential equations. The discretization provided a useful and consistent approximation to the solutions of dimensionless governing equations. Hence, the governing equations (10) - (12) based on the boundary conditions (13) and (14) are discretized with the step size of $h = 0.001$ with the given boundary conditions. The resulting nonlinear system of algebraic equations is solved by employing the modified Newton's method to obtain the results. The numerical computations are transformed into numerical codes using MAPLE application software as discussed by Heck [34]. Hence, the finite difference equations corresponding to the couple governing equations (10) – (12) are given as:

$$\begin{aligned} f_{i+2} &= 2f_{i+1} + 2f_{i-1} - f_{i-2} \\ &- 2hf_i\left(f_{i+1} - 2f_i + f_{i-1}\right) + \frac{1}{2}\left(f_{i+1} - f_{i-1}\right)^2 \\ &- 2h^3\varepsilon^2 + \frac{M}{2}\left[4h^3\varepsilon - 2h^2\left(f_{i+1} - f_{i-1}\right)\right] \\ &+ h^2P\left(f_{i+1} - f_{i-1}\right) \end{aligned} \tag{15}$$

$$\theta_{i+1} = \frac{h\Pr f_i - 2}{2 + h\Pr f_i}\theta_{i+1} + \frac{\left\{\left[2h^2\Pr(R+1) + H\right] + 4\right\}}{2 + h\Pr f_i}\theta_i \tag{16}$$

$$\phi_{i+1} = \frac{2h^2\gamma + 4}{2 + hScf_i}\phi_i^n - \frac{2 - hScf_i}{2 + hScf_i}\phi_{-1i} \tag{17}$$

IV. Results and Discussion

Numerical solutions are obtained for the velocity f, temperature θ and species concentration φ profiles for different values of governing parameters. Results are displayed through graphs in Figs 1-14. It should be noted that Figs. (1 – 4) satisfy the specified boundary conditions and Figs. (5 – 14) reveal that the far field boundary conditions are satisfied asymptotically and hence this supports the accuracy of the numerical computations and results. Tables 1 and 2 display the local wall stress, rates of heat and mass transfer at the surface of the stretching sheet for some results of physical interests in engineering.

The velocity profiles f for different values of magnetic interaction parameter M, suction parameter S, stretching ratio parameter ε and permeability parameter P are displayed in Figs. 1-4 respectively. Fig. 1 shows the influence of M on the flow distribution. It is evident from this fig. that f is an increasing function of M. This corresponds to a decrease in momentum boundary layer thickness. Fig. 2 depicts the effect S on f. It is obvious that an increase in the values of S increases the boundary layer thickness in the flow distribution. Fig. 3 illustrates the influence of stretching ratio ε on f. An increase in the values of ε is observed to increase the velocity profile. The effect of permeability parameter P on f is depicted in Fig. 4. f is found to be a decreasing function of P.

Figs. 5 - 9 illustrate the variations of θ with respect to η for various values of ε, Pr, S, R and H respectively. From Fig. 5, it is observed that ε has a decreasing effect on θ. The feature of Pr on θ is displayed in Fig. 6. Pr is a decreasing function of the temperature profile. Higher estimation of Pr is found to decay the temperature distribution in the flow regime. This is due to the fact that Pr expresses the ratio of thermal diffusivity to momentum diffusivity. Thus, small values of Pr implies that the thermal diffusivity dominates and for higher estimation of Pr, the momentum diffusivity dominates. Fig. 7 expresses the effect of S on the temperature profile. Suction velocity parameter is also found to reduce the fluid temperature. The influence of thermal radiation R on θ is portrayed by Fig. 8. The thermal boundary layer thickness decreases as the values of R increases. As the values of H increase, the temperature profile increases for the heat generation/absorption parameter as shown in Fig. 9. Positive values of H corresponds to heat generation whereas negative values of H signifies heat absorption. Though, when heat generation is considered in the flow regime, the fluid temperature is higher than that of heat absorption. Consequently, the thermal boundary layer thickness and the surface temperature are enhanced by increment in the values of H.

Figs. 10 - 13 describe the influences of γ, ε, Sc and n on the species concentration profile. Figs. 10 captures the influence of γ on the species concentration profile. It is observed that for any given value of η, the species concentration becomes decreased with an increase in γ. The variation of φ with different values of ε is indicated by Fig. 11. It is clear that stretching ratio parameter decreases the species concentration profile. The effect of suction parameter on the species concentration profile is shown in Fig. 12. It is obvious that an increase in the values of suction parameter decreases the species concentration boundary layer thickness. Fig. 13 is plotted to display the influence of Sc on species concentration distribution in the boundary layer. It is observed that increasing the values of Sc corresponds to an decrease in the species concentration boundary layer thickness. Lastly, Fig. 14 is plotted to show the influence of n on the species concentration profile. The species concentration boundary layer thickness is found to increase with increment in the values of the order of chemical reaction parameter.

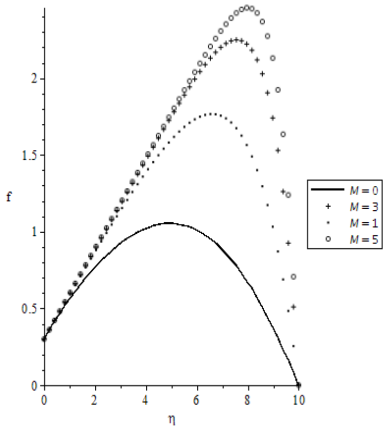

Fig. 1. Influence of M on f.

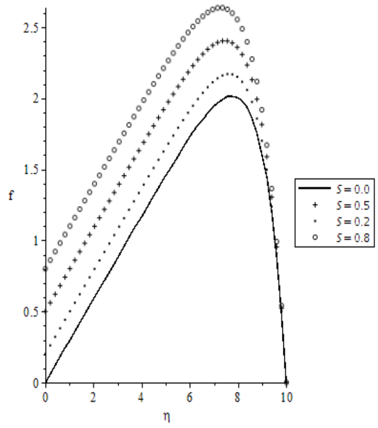

Fig. 2. Influence of S on f.

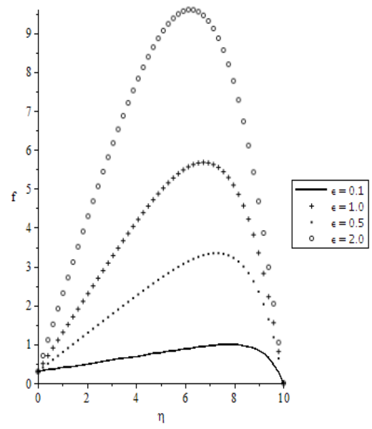

Fig. 3. Influence of ε on f.

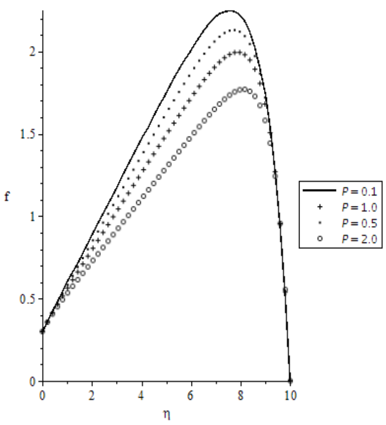

Fig. 4. Influence of P on f.

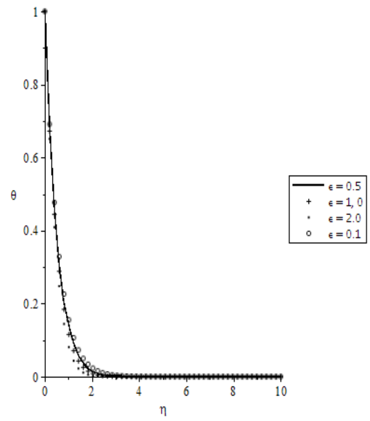

Fig. 5. Influence of M on θ.

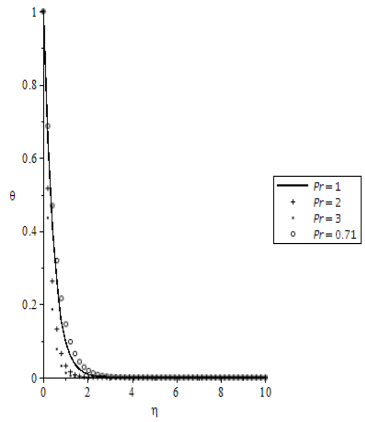

Fig. 6. Influence of Pr on θ.

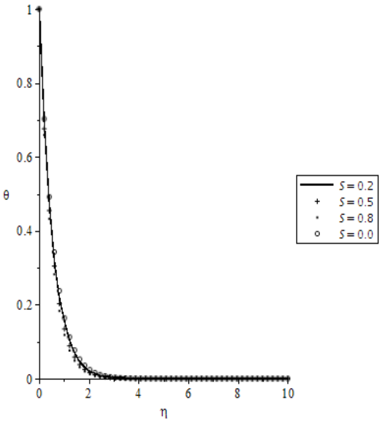

Fig. 7. Influence of S on θ.

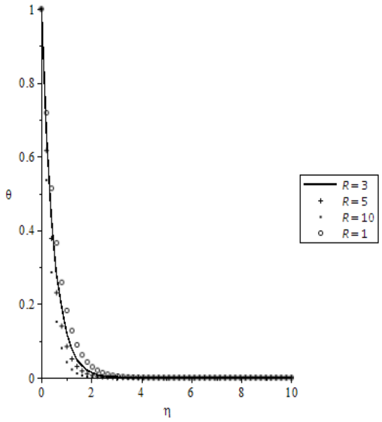

Fig. 8. Influence of R on θ.

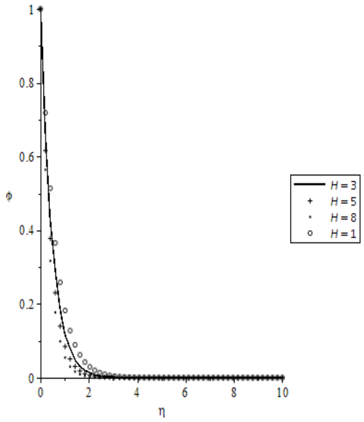

Fig. 9. Influence of H on θ.

Table 1. Numerical experiment showing $-f''(0)$, $-\theta'(0)$ and $-\phi(0)$ at $Rt = 2$, $M = 3$, $\varepsilon = 0.5$ Pr = 0.71 and $P = 2$.

M	P	ε	Pr	Rt	$-f''(0)$	$-\theta'(0)$	$-\phi'(0)$
5					3.8649384552	0.2891933715	0.7378377852
8					4.6973428508	0.9416096230	0.7217633972
10					5.1753268885	1.4775669733	0.7147344795
	1				3.7577154509	0.8176371544	0.7339600069
	2				3.8649384552	0.2891933715	0.7378377852
	3				3.9767610404	0.0155509104	0.7406579646
		0.5			3.8649384552	0.2891933715	0.7378377852
		0.6			4.0203202664	0.8810675154	0.7299167258
		0.7			4.1684394996	2.7367492117	0.7227757933
			0.5		3.8649384552	1.1718641494	0.7378377852
			0.6		3.8649384552	0.6032771101	0.7378377852
			0.7		3.8649384552	0.3138677521	0.7378377852
				2.0	3.8649384552	0.2891933715	0.7378377852
				2.1	3.8649384552	0.6761867897	0.7378377852
				2.3	3.8649384552	7.1661018942	0.7378377852

Table 2. Numerical experiment show $-f''(0)$, $-\theta'(0)$, and $-\phi(0)$ at $S = 0.3$, $Sc = 0.62$, $H = 3$, $\gamma = 1$ and $n = 1$.

H	Sc	γ	n	S	$-f''(0)$	$-\theta'(0)$	$-\phi(0)$
1					3.8649384552	0.9945449601	0.7378377852
3					3.8649384552	0.2891933715	0.7378377852
5					3.8649384552	0.1114886139	0.7378377852
	0.78				3.8649384552	1.0525006503	1.0525006503
	0.94				3.8649384552	0.2891933715	1.1787772997

H	Sc	γ	n	S	$-f''(0)$	$-\theta'(0)$	$-\phi(0)$
	2.62				3.8649384552	0.2891933715	2.2494482535
		0			3.8649384552	0.2891933715	0.0044787365
		1			3.8649384552	0.2891933715	0.7378377852
		2			3.8649384552	0.2891933715	1.0118146308
			1		3.8649384552	0.2891933715	0.9170554253
			2		3.8649384552	0.2891933715	0.7378377852
			4		3.8649384552	0.2891933715	0.5605695380
				0.1	3.7278165372	0.4883396894	0.6855812181
				0.3	3.8649384552	0.2891933715	0.7378377852
				0.5	4.0069430966	0.1578602934	0.7953915675

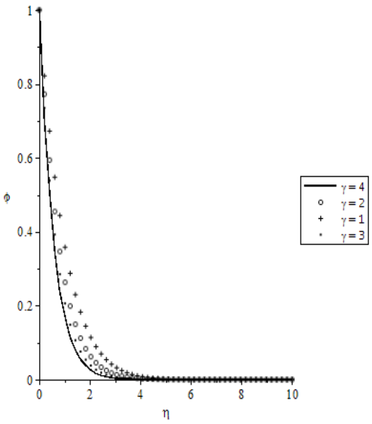

Fig. 10. Influence of γ on ϕ.

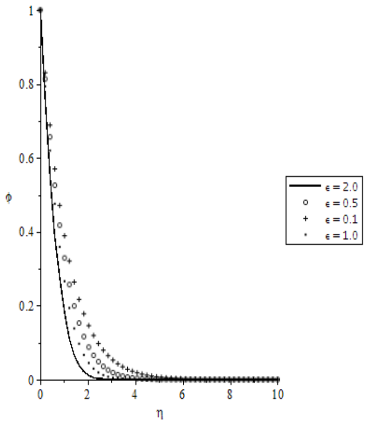

Fig. 11. Influence of ε on ϕ.

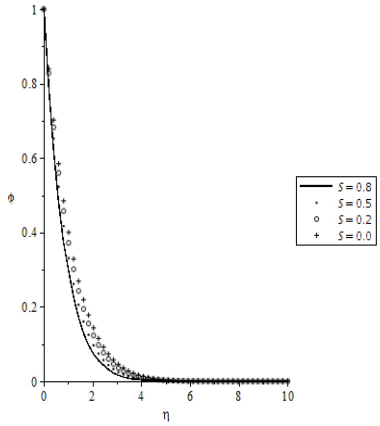

Fig. 12. Influence of ε on ϕ.

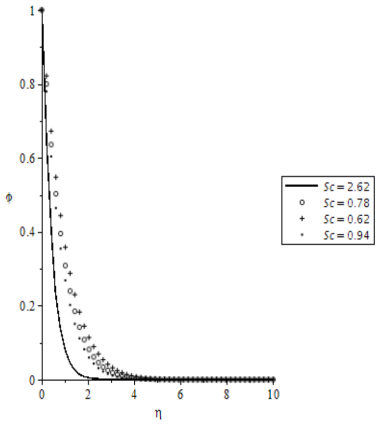

Fig. 13. Influence of Sc on ϕ.

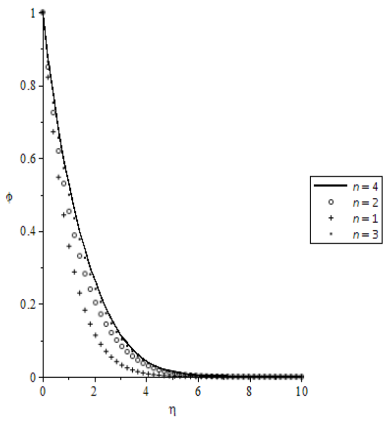

Fig. 14. Influence of n on ϕ.

V. Conclusion

The present study proposes a numerical approach of finite difference technique with modified Newton's method to solve the problem of the heat and mass transfer of a magnetohydrodynamic reactive flow of a viscous fluid over a stretching sheet in the presence of thermal radiation, internal heat generation/absorption and chemical reaction of order n. The accuracy of the numerical computations and results is supported by the velocity profiles satisfying the specified boundary conditions while both temperature and species concentration profiles establish that the far field boundary conditions are asymptotically satisfied. The results of emerging physical parameters on the velocity, temperature and species concentration profiles are presented, discussed through plots and are summarized as follows: (i) he velocity distribution increases for higher estimation of suction, velocity ratio and magnetic interaction parameters but it decays with increasing values of the permeability parameter; (ii) the temperature distribution is enhanced by increasing values of both heat generation/ absorption and suction parameters. However, the temperature is a decreasing function of Prandtl number, stretching ratio and thermal radiation parameter; (iii) the species concentration distribution increases by increasing values of order of chemical reaction parameter whereas the species concentration is found to decay with increases in the values of Schmidt number, rate of chemical reaction, stretching ratio and suction parameters; (iv) The local wall stress is enhanced with the increasing values of Hartmann number, permeability suction and stretching rate parameters; increasing values of the rate of chemical reaction, permeability, heat generation and thermal radiation parameters with those of Prandtl and Schmidt numbers increase the local rate of heat transfer while the order of chemical reaction, rate of chemical reaction and thermal radiation parameters boost the coefficient of mass transfer on the stretching sheet.

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