International Journal of Applied Science and Mathematics
Volume 6, Issue 5, ISSN (Online): 2394-2894

Multilayer Partial Differential Heat Transfer Model for Temperature of Each Layer of Thermal Protective Clothing Under High Temperature Operation

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Date of publication (dd/mm/yyyy): 27/10/2019

Abstract **– Thermal protective clothing is a kind of protective clothing that protects people working under high temperature or ultra-high temperature conditions, so as to avoid damage to the human body caused by heat source. Therefore, it is helpful to grasp the temperature change outside the skin when wearing thermal protective clothing. For better modification of thermal protective clothing. In this paper, the mathematical model is used to determine the temperature change outside the skin of the dummy. The temperature variation of the external temperature and the internal temperature is different, the thickness is different, and the temperature is changed under different time conditions. The related partial differential mathematical model is established. Generally, there is a heat exchange equation between the air layer and the human body. When the skin temperature reaches a certain temperature, it will reach thermal equilibrium with the body. This paper uses COMSOL simulation to obtain the key transfer function, and then uses the MATLAB toolbox to make a three-dimensional surface, and then uses the difference method. Calculate the corresponding numerical solution, and finally use COMSOL to verify the results, and obtain the temperature changes of each layer of the thermal protective suit.**

Keywords **– Partial Differential Equation, Heat Conduction, Difference Method, MATLAB, COMSOL.**

I. Introduction

When working in a high temperature environment, people need to wear special clothing to avoid burns. Special clothing is usually composed of three layers of materials, which are recorded as layers I, II and III. The layer I is in contact with the external environment, and there is a gap between the layer III and the skin. This gap is recorded as an IV layer, which is an air layer. In order to reduce R&D costs and shorten the development cycle, mathematical models are used to determine the temperature changes outside the skin of the dummy.

The dummy whose body temperature was controlled at 37 ℃ was placed in a high temperature environment of the laboratory, and the temperature outside the skin of the dummy was measured. The discussion was carried out at an ambient temperature of 75 ℃, a second layer thickness of 6 mm, a fourth layer thickness of 5 mm, and a working time of 90 minutes.

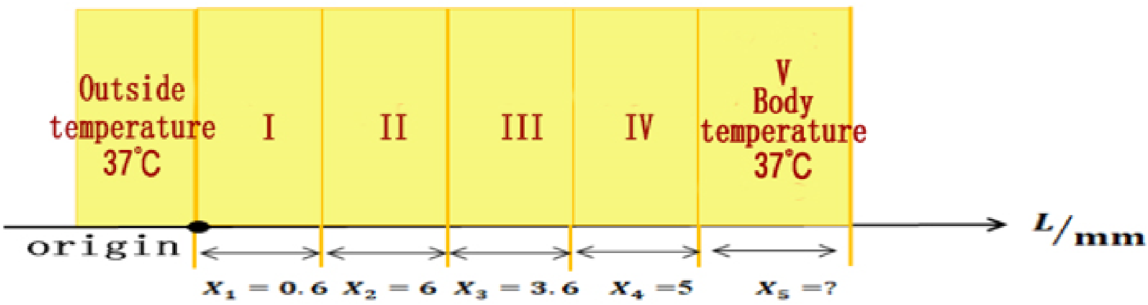

Fig. 1. Schematic diagram of the thermal protective clothing structure.

Table 1. Thermal protective clothing parameter information.

Layering	Density (kg/ m^3)	Specific heat (*J*/ (*kg* . ℃))	Thermal conductivity (*W*/ (m · ℃))	Thickness (mm)
I	300	1377	0.082	0.6
II	862	2100	0.37	0.6-25
III	74.2	1726	0.045	3.6
IV	1.18	1005	0.028	0.6-6.4

II. Establishment of Multi-Layer Heat Conduction Partial Differential Equation Model

This paper uses partial differential equations and Fourier transform to solve the heat conduction problem [1] [2], for a multi-layer, a complete mathematical model can be built by separately calculating and considering the boundary conditions, and calculations are performed by MATLAB software. The air layer and the human body (the V layer) have a heat exchange equation inside, and when the skin temperature reaches 48.08 ° C, the heat balance is reached with the body [3].

It is assumed that the parameter C is the sensible heat capacity of each layer medium; k is the thermal conductivity of each layer medium; θ_i is the temperature; t is the time; x is the horizontal coordinate; Q is the energy; F_L, F_R is the left-to-right radiation amount.

We only consider the heat conduction and heat radiation between the layers, so we have established a one-dimensional mathematical model of partial differential equations:

$$C\ \frac{\partial\theta_i}{\partial t}=\frac{\partial}{\partial x}\left(k\ \frac{\partial\theta_i}{\partial x}\right),(x,t)\in\Omega_1\times\left(0,t_{\exp}\right) \tag{1}$$

The conduction model of different layers should be brought into different solutions according to the known parameters. At the interface of different layers, we need to consider the boundary conditions of different layers, which conform to the law of conservation of energy;

$$-k\frac{\partial\theta_i}{\partial x}|_{x=0}=Q; \tag{2}$$

$$-k\frac{\partial T}{\partial x}|_{x=L}=Q; \tag{3}$$

Thermal Conductivity Model for Layer I:

$$C_I\frac{\partial\theta_1}{\partial t}=\frac{\partial}{\partial x}\left(k_1\frac{\partial\theta_1}{\partial x}\right),x\in\left[0,L_1\right] \tag{4}$$

$$k_1\frac{\partial\theta_1}{\partial x}|_{X=0}=\left(q_{con}+q_{rad}\right)|_{x=0} \tag{5}$$

Heat Transfer Model of the Second Layer:

$$C_{II}\frac{\partial\theta_2}{\partial t}=\frac{\partial}{\partial x}\left(k_2\frac{\partial\theta_2}{\partial x}\right), x\in[L_1,L_2] \tag{6}$$

$$-k_2\frac{\partial\theta_1}{\partial x}|_{X=L1}=-k_1\frac{\partial\theta_2}{\partial x}|_{x=L1} \tag{7}$$

Heat Transfer Model of the Third Layer:

$$C_{III}\frac{\partial\theta_3}{\partial t}=\frac{\partial}{\partial x}\left(k_3\frac{\partial\theta_3}{\partial x}\right), x\in[L_2,L_3] \tag{8}$$

$$-k_3\frac{\partial\theta_3}{\partial x}|_{X=L1+L2}=-k_2\frac{\partial\theta_2}{\partial x}|_{x=L1+L2} \tag{9}$$

Heat Transfer Model of Layer IV:

$$\left(\rho c_p\right)_{air}\frac{\partial\theta_4}{\partial t}=\frac{\partial}{\partial x}\left(k_{air}\frac{\partial T}{\partial x}\right), x\in[L_3,L_4] \tag{10}$$

$$-k_3\frac{\partial\theta_4}{\partial x}|_{X=L3}=\left(q_{air,rad}-k_{air}\frac{\partial\theta_4}{\partial x}\right)|_{x=L4} \tag{11}$$

Heat Transfer Model of Layer V:

$$C_V\frac{\partial\theta_5}{\partial t}=\frac{\partial}{\partial x}\left(k_5\frac{\partial\theta_5}{\partial x}\right), x\in[L_4,L_5] \tag{12}$$

$$-k_4\frac{\partial\theta_4}{\partial x}|_{X=L4+L5}=-k_5\frac{\partial\theta_5}{\partial x}|_{x=L4+L5} \tag{13}$$

Thus the Overall Model:

Heat conduction equation

$$\begin{cases} C_I\frac{\partial\theta_1}{\partial t}=\frac{\partial}{\partial x}\left(k_1\frac{\partial\theta_1}{\partial x}\right), x\in(0,L_1) \\ C_{II}\frac{\partial\theta_2}{\partial t}=\frac{\partial}{\partial x}\left(k_2\frac{\partial\theta_2}{\partial x}\right), x\in(L_1,L_2) \\ C_{III}\frac{\partial\theta_3}{\partial t}=\frac{\partial}{\partial x}\left(k_3\frac{\partial\theta_3}{\partial x}\right), x\in(L_2,L_3) \\ \left(\rho c_p\right)_{air}\frac{\partial\theta_4}{\partial t}=\frac{\partial}{\partial x}\left(k_{air}\frac{\partial\theta_4}{\partial x}\right)-\frac{\partial q_{air,rad}}{\partial x}, x\in(L_3,L_4) \\ C_V\frac{\partial\theta_5}{\partial t}=\frac{\partial}{\partial x}\left(k_5\frac{\partial\theta_5}{\partial x}\right), x\in(L_4,L_5) \end{cases} \tag{14}$$

Boundary conditions

$$\begin{cases} -k_1 \dfrac{\partial \theta_1}{\partial x} |_{X=0} = \left(q_{con} + q_{rad} \right) |_{x=0} \\ -k_2 \dfrac{\partial \theta_2}{\partial x} |_{X=L1} = -k_1 \dfrac{\partial \theta_1}{\partial x} |_{x=L1} \\ -k_3 \dfrac{\partial \theta_3}{\partial x} |_{X=L1+L2} = -k_2 \dfrac{\partial \theta_2}{\partial x} |_{x=L1+L2} \\ -k_3 \dfrac{\partial \theta_3}{\partial x} |_{X=L3} = \left(q_{air,rad} - k_{air} \dfrac{\partial \theta_3}{\partial x} \right) |_{x=L4} \\ -k_4 \dfrac{\partial \theta_4}{\partial x} |_{X=L4+L5} = -k_5 \dfrac{\partial \theta_5}{\partial x} |_{x=L4+L5} \end{cases} \tag{15}$$

Initial Value Condition:

$$\theta_1 = \theta_2 = \theta_3 = \theta_4 = \theta_5 = 37\ {}^0C \tag{16}$$

III. Parameter Calibration of Multilayer Thermal Conduction Partial Differential Equation Mode

(1) The known parameters in the model are the thermal conductivity K of the I, II, III, IV layers, the specific heat C, and the thickness L_1, L_2, L_3, L_4 of each layer, the thickness L_5 of the V layer is unknown, and the heat transfer coefficient A of the V layer;

(2) Using experimental data, let $Date_i = \{date_1 + date_2 + ... + date_{5400}\}$, where $Date_i$ represents the temperature value per second for a total of 90 minutes, so i takes 5400s, Find the temperature equation of the VI layer $Out = \theta_4(x_5, t)$; Then use the least squares method to determine the key parameters [4];

$$\min \sum_{i=1}^{5400} \left(out_i - date_i \right) \tag{17}$$

Where $k_5 \in (1,8), L_5 \in (0,20)$ 。

Then get: $k_5 = 4.3, L_5 = 12.5mm$;

Get simulation images based on COMSOL and MATLAB values [5] [6]:

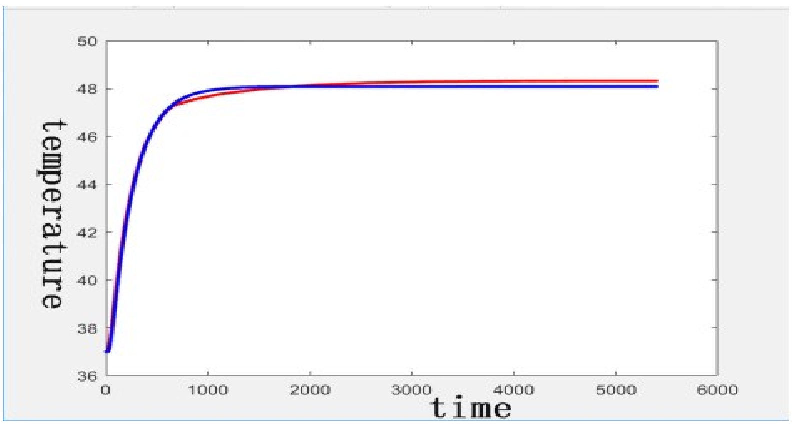

Fig. 2. Curves obtained using COMSOL simulation and fitting curves for finding parameters.

Through the fitting of the image, it can be found that the correlation between the two curves is good, and the calculation parameters of the fifth layer are effective.

According to the establishment of the above model, a multi-layer partial differential heat conduction model of the temperature of each layer of the thermal protective suit under high temperature operation can be obtained: Heat conduction equation

$$\begin{cases} 1377\dfrac{\partial\theta_1}{\partial t}=\dfrac{\partial}{\partial x}\left(0.082\dfrac{\partial\theta_1}{\partial x}\right), x\in(0,0.6) \\ 2100\dfrac{\partial\theta_2}{\partial t}=\dfrac{\partial}{\partial x}\left(0.37\dfrac{\partial\theta_2}{\partial x}\right), x\in(0.6,6.6) \\ 1726\dfrac{\partial\theta_3}{\partial t}=\dfrac{\partial}{\partial x}\left(0.045\dfrac{\partial\theta_3}{\partial x}\right), x\in(6.6,10.2) \\ 1.18\times1005_{air}\dfrac{\partial\theta_4}{\partial t}=\dfrac{\partial}{\partial x}\left(0.028\dfrac{\partial\theta_4}{\partial x}\right)-\dfrac{\partial q_{air,rad}}{\partial x}, x\in(10.2,15.2) \\ 1005\dfrac{\partial\theta_5}{\partial t}=\dfrac{\partial}{\partial x}\left(0.036\dfrac{\partial\theta_5}{\partial x}\right), x\in(15.2,27.7) \end{cases} \tag{18}$$

Boundary Conditions

$$\begin{cases} -0.082\dfrac{\partial\theta_1}{\partial x}|_{X=0}=\left(q_{con}+q_{rad}\right)|_{x=0} \\ -0.37\dfrac{\partial\theta_2}{\partial x}|_{X=0.6}=-0.082\dfrac{\partial\theta_2}{\partial x}|_{0.6} \\ -0.045\dfrac{\partial\theta_3}{\partial x}|_{X=12}=-0.37\dfrac{\partial\theta_3}{\partial x}|_{x=12} \\ -0.045\dfrac{\partial\theta_4}{\partial x}|_{X=3.6}=\left(q_{air,rad}-0.028\dfrac{\partial\theta_4}{\partial x}\right)|_{x=5} \\ -0.028\dfrac{\partial\theta_5}{\partial x}|_{X=17.5}=-0.036\dfrac{\partial\theta_5}{\partial x}|_{x=17.5} \end{cases} \tag{19}$$

IV. CONCLUSION

According to the temperature curve of each layer of the thermal protective clothing obtained by the experiment, it is found that the temperature of each layer boundary reaches a stable state after a certain time. At the same time, the temperature distribution of different time points is simulated by MATLAB. Finally, it is proposed. Schematic diagram of the temperature of each interface when combined for 60 minutes:

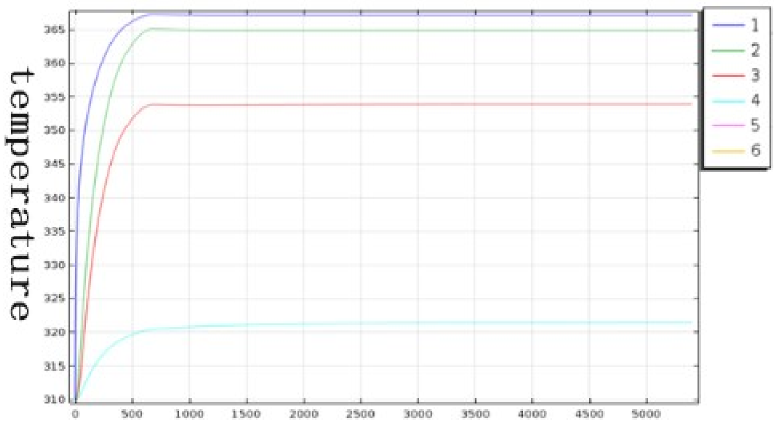

Fig. 3. Temperature curve on the boundary of each layer.

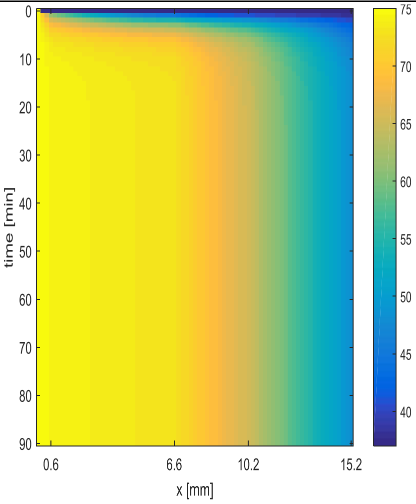

Fig. 4. Temperature distribution heat map under MATLAB simulation.

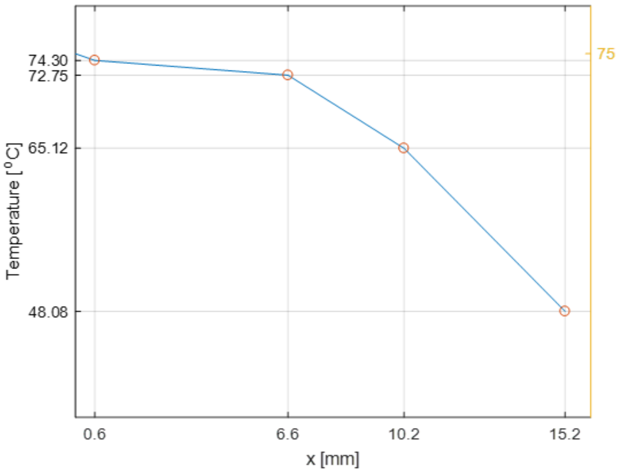

Fig. 5. Schematic diagram of the temperature of each interface at 60 min.

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