International Journal of Applied Science and Mathematics
Volume 7, Issue 5, ISSN (Online): 2394-2894

Modelling of Water Pollutants with Weibull and Lognormal Distribution: A Case Study of Oyo State, Nigeria

Saheed A. Afolabi
Department of Statistics, University of Ibadan, Ibadan, Nigeria.
Corresponding author email id: Afsab4ever@yahoo.com

Date of publication (dd/mm/yyyy): 04/10/2020

***Abstract* – Water is considered polluted when unwanted materials with potentials to threaten human and other natural systems find their ways into water sources (Rivers, lakes, well, boreholes or reserved fresh water) in homes or industries. In this work, an attempt is made to use different statistical techniques such as preliminary study, data visualization, hypothesis testing with respect to the standards by the World Health Organization and the Standards Organization of Nigeria through modelling distribution and goodness of fit to 13 pollutants of two different rivers in Oyo state to assess its safety and it was examined that not all the pollutants were right skewed. The pairwise correlation showed that few of the pollutants were highly correlated and they exhibit trend which claimed further corresponding testing which established the concentration of Pollutants to be higher in Eleyele than Asejire for almost all Pollutants. Lognormal distribution described the pattern of the various pollutants better than Weibull distribution through the goodness of fit application.**

***Keywords* – Modelling, Lognormal Distribution, Weibull Distribution, Goodness of Fit.**

I. Introduction

Pollution is simply the contamination of the Earth’s environment with materials that interfere with human health, the quality of life and the natural functioning of the ecosystem. Although, some environmental pollution is as a result of natural causes such as volcanic eruptions but the major cause of pollution in Nigeria is man-made. Water is considered polluted when unwanted materials with potentials to threaten human and other natural systems find their ways into water sources (Rivers, lakes, well, boreholes or reserved fresh water) in homes or industries. The Water pollution affects plants and organisms living in these bodies of water and the contamination of water bodies is one of the important environmental problems worldwide. Water pollution has become a very serious environmental challenge in Nigeria, particularly in the cities where there are lots of industries while Nigeria is not left out in this global problem [Aladesanmi et al., (2013), Arojojoye and Adeosun, (2016) and Ipinmoroti, (2013)]. Most industries in Nigeria discharge their waste products directly into rivers, rendering the rivers polluted, this research therefore aimed at modelling water pollutants of a particular river “Asejire” in southwestern part of Nigeria using two rightly skewed distributions.

Asibor *et al.*, (2015) evaluated the heavy metal content of the Asejire reservoir. Twenty stations were selected, sampled and analyzed using standard methods. Standard pollution indices such as Enrichment Factor, Contamination Factor and Pollution Load Index were deployed to assess the level of heavy metals contamination in the reservoir.

Obisesan *et al.*, (2012) attempted to use environmental pollution data to explain data visualization with respect to the standards by the World Health Organization (W.H.O.) and the Standard Organization of Nigeria. They discussed exploratory data analysis and change point detection. Their effort is aimed at achieving some methodological changes in the approach to analyzing environmental data such that statistical methodologies co-

-ould be enhanced for sustainable development.

Lameed and Obadara (2006) studied the impact of eco-development impact of Coca-Cola industry on biodiversity resources at Asejire area in Ibadan, Nigeria. They collected water samples from the river and the parameters analyzed were dissolved oxygen concentration, pH, temperature, turbidity and some heavy metals. They indicated that the water is acidic and might be toxic for human consumption except been treated to comply with WHO standard.

II. Methodology

2.1. *Exploratory Data Analysis*

Minimum

$$\min(X) = x \in X \ni x \le x_i \forall i \in n(X) \tag{1}$$

Maximum

$$\max(X) = x \in X \ni x \ge x_i \forall i \in n(X) \tag{2}$$

Range

$$\text{range(X)} = \text{max(X)} - \text{min(X)} \tag{3}$$

Mean

$$\overline{\text{X}} = \frac{\sum_{i=1}^{n} X_i}{n} \tag{4}$$

Standard Deviation

$$\sigma = \sqrt{\frac{\sum_{i=1}^{n} (X_i - \overline{\text{X}})^2}{n}} \tag{5}$$

Skewness

$$\gamma_1 = \frac{m_3}{\sigma^3} \tag{6}$$

where m_3 is the third-order moment of the data given by:

$$m_3 = \frac{\sum_{i=1}^{n} (X_i - \overline{\text{X}})^3}{n} \tag{7}$$

And $\overline{\text{X}}$ is the arithmetic mean, n is the number of total observation and σ^3 is the standard deviation raised to the third power with X being the pollutant of interest.

2.2. *Data Visualization*

- Time plots of pollutants from both rivers for possibility and comparison of trend or breakpoint, if any exist-

-s.

- Density plots for check of plausible distribution of pollutants.
- Bar plot for comparison of means of pollutants in the two rivers.

2.3. *Test of Pollutants for Conformance to Standards*

That water intended for human consumption must be free from chemical substances and microbials in amount which would produce a harmful effect is universally unarguable. Supplies of drinking water should not only be as aesthetically attractive as possible. Absence of turbidity, colour and detectable and disagreeable taste and odours is important in water supplies intended for domestic use, (W.H.O., 2006). In this research, interest is on comparing observed components of water sample collected from the two rivers to check for conformance to best practice. Here, the following test of hypothesis was conducted.

2.3.1. *Test for Quality on Standards with Interval*

H_0: Pollutant level is within standard.

H_a: Water is polluted with pollutant.

Test Statistic

The test statistic used here is the range (X) where X is the pollutant of interest.

Decision

Reject $H0$ if $x \in R(X) \ni x \notin S(X)$ and conclude appropriately or reject $H0$ if $R(X) \subsetneq S(X)$ where $R(X)$ is the range of the pollutant and $S(X)$ is the W.H.O. standard for the pollutant.

2.3.2. *Comparison of Pollutant Means*

Having ascertained pollutants which were out of standards for the two rivers, another question to be answered is ‘which of the two rivers is more polluted’? This question is answered with the following test of hypothesis:

H_0: $\mu_1 = \mu_2$

H_a: $\mu_1 \neq \mu_2$

Tested at 5% level of significance using the standard normal test statistic.

2.4. *Modelling the Distribution of Pollutants*

Weibull Distribution

The probability density function of weibull for the water pollutants (x), with two parameters α and β is given by:

$$f(\text{x}, \alpha, \beta) = \frac{\alpha}{\beta}\left(\frac{x}{\beta}\right)^{\alpha-1} \exp\left[-\left(\frac{x}{\beta}\right)^{\alpha}\right]; 0 < x < \infty \quad (8)$$

Where α , β and x are the shape parameter, scale parameter and water pollutants value respectively.

Log-Normal Distribution

The probability density function of log-normal for water pollutants (x), with two parameters μ and σ is given by:

$$f(x,\mu,\sigma)=\frac{1}{x\sigma\sqrt{2\pi}}\exp\left\{-\frac{1}{2}\left(\frac{\ln(x)-\mu}{\sigma}\right)^2\right\};0<x<\infty \quad (9)$$

Where σ , μ and x are the shape parameter, scale parameter and water pollutants respectively.

2.5. *Goodness of Fit (GOF) Test*

Goodness of fit tests indicate whether or not it is reasonable to assume that a random sample comes from a specific distribution. They are a form of hypothesis testing where the null and alternative hypotheses are:

H_0 : Sample data came from a stated distribution.

H_1 : Not H_0.

The Chi-square test is the oldest goodness of fit test dating back to (Pearson, 1900). The test may be thought of as a formal comparison of histogram with fitted density.

Test of Hypothesis

The chi-square test is defined for the hypotheses:

Hypotheses

$H_01 : X \sim lognormal(\mu; \sigma^2)$

$H_11 : not\ H_01$

also,

$H_02 : X \sim Weibull(\alpha; \beta)$

$H_12 : not\ H_02$

where X represents amount of pollutant. Test Statistic and Critical value.

$$\chi^2_{cal} = \sum\left(\frac{(o-E)^2}{E}\right) \quad (10)$$

$$\chi^2_{tab} = \chi^2_{(n-1),1-\frac{\alpha}{2}} \quad (11)$$

Where O is the observed frequency and E is the expected frequency.

Decision and Conclusion

Reject null hypothesis if p - *value* is less than a significance level of α and conclude appropriately.

III. Results and Discussions

This section gives the properties of the two rivers in terms of lognormal and Weibull distributions based on the pollutants' skewness.

Table 1. Preliminary analysis of Asejire River.

Pollutants	Min	Max	Mean ± sd	Skewness
Turbidity	1.0	4.0	2.68 ± 0.81	-0.70
Colour	4.0	7.0	5.03 ± 0.41	1.69
pH	6.4	8.8	7.49 ± 0.54	0.66
Dissolved Oxygen	6.4	11.0	7.60 ± 0.87	1.70
Alkalinity	22.0	78.0	38.68 ± 9.37	1.41
Total Hardness	36.0	88.0	57.03 ± 10.35	0.32
CaH	9.0	70.0	38.7 ± 10.62	-0.18
Chloride	10.4	35.0	19.45 ± 5.59	1.04
Iron	0.2	0.3	0.24 ± 0.02	-0.29
Silica	4.0	14.0	10.33 ± 2.74	-0.38
Sol	84.0	217.0	138.9 ± 25.95	0.37
Dissolved Solids	48.0	190.0	109.5 ± 26.61	0.05
Solid Suspensions	12.0	86.0	39.75 ± 14.76	1.27

The Table 1 affirmed the positive skewness of the majority water pollutants at Asejire river of Ibadan, Oyo State except the Turbidity, CaH, Iron and Silica.

Table 2. Preliminary analysis of Eleyele River.

Pollutants	Min	Max	Mean ± sd	Skewness
Turbidity	2.0	32.9	8.24 ± 5.61	2.02
Colour	5.0	25.0	7.62 ± 4.17	2.26
pH	6.2	8.0	7.19 ± 0.39	-0.51
Dissolved Oxygen	5.5	54.0	11.20 ± 9.70	3.72
Alkalinity	8.0	100.0	48.37 ± 17.14	0.77
Total Hardness	64.0	112.0	90.50 ± 10.82	-0.29
CaH	34.0	100.0	65.47 ± 12.06	0.44
Chloride	23.5	56.8	36.22 ± 5.81	1.21
Iron	2.0	2.6	2.33 ± 0.18	-0.12
Silica	4.0	17.0	12.88 ± 2.59	-1.24
Sol	178.0	365.0	246.8 ± 28.26	1.62
Dissolved Solids	138.0	245.0	173.4 ± 14.77	1.89

Pollutants	Min	Max	Mean ± sd	Skewness
Solid Suspensions	36.0	92.0	69.92 ± 10.93	-1.30

The above Table 2 affirmed the positive skewness of the majority water pollutants at Eleyele River of Ibadan, Oyo State.

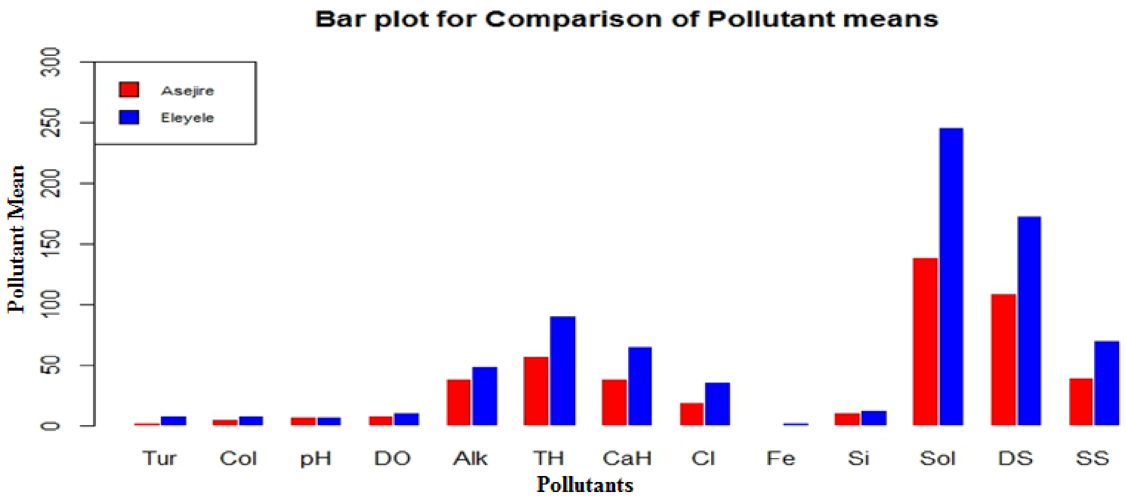

Fig. 1. Pollutants' bar plot of asejire and Eleyele.

The Fig. 1 above gives the means' comparison of the two major rivers (Asejire and Eleyele). It is established that pollutant means of Eleyele river are more than the pollutant means of Asejire river.

Table 3. Asejire test of Conformance.

Pollutants	Parameter	W.H.O Standard	Conformance
Turbidity	1.0 - 4.0	(1.0 - 5.0) NTU	Yes
Color	4.0 - 7.0	(3.0 -15.0) TCU	Yes
pH	6.4 - 8.8	6.5- 9.5	**No**
D.O.	6.4 - 11.00	*no health-based guideline*	Indecisive
Alk	78.0	< 100 *mg/l*	Yes
TH	36.0- 88.0	(100 - 500) *mg/l*	**No**
CaH	9.0 - 70.0	*no health-based guideline*	Indecisive
Cl	10.4 - 35.0	(200 - 250) *mg/l*	**No**
Iron	0.21 - 0.28	(0.1 - 0.3) *mg/l*	Yes
Silica	4.0 - 14.0	*no health-based guideline*	Indecisive
Sol	84.0 - 217.0	*no health-based guideline*	Indecisive
DS	48.0 - 190.0	(500 - 600) *mg/l*	**No**
SS	12.0 - 86.0	*no health-based guideline*	Indecisive

Source: WHO, (2006) and Saheed A. A., (2019).

It can be affirmed from the above Table 3 that the Asejire River contained the following Pollutants (pH, Cl, TH and DS) which are at intolerable level.

Table 4. Eleyele test of Conformance.

Pollutants	Parameter	W.H.O Standard	Conformance
Turbidity	2.0 - 12.0	(1.0 - 5.0) NTU	**No**
Color	5.0 - 25.0	(3.0 -15.0) TCU	**No**
pH	6.2 - 8.0	6.5- 9.5	**No**
D.O.	5.5 - 54.0	*no health-based guideline*	Indecisive
Alk	8.0 - 100.0	< 100 *mg/l*	Yes
TH	64.0 - 112.0	(100 - 500) *mg/l*	**No**
CaH	34.0 - 100.0	*no health-based guideline*	Indecisive
Cl	23.5 - 56.8	(200 - 250) *mg/l*	**No**
Iron	2.0 - 2.6	(1.0 - 3.0) *mg/l*	Yes
Silica	4.0 - 17.0	*no health-based guideline*	Indecisive
Sol	178.0 - 365.0	*no health-based guideline*	Indecisive
DS	138.0 - 245.0	(500 - 600) *mg/l*	**No**
SS	36.0 - 92.0	*no health-based guideline*	Indecisive

In the same vein, Table 4 above established that the Eleyele River contained the Pollutants (Turbidity, Color, pH, TH, Cl and DS) which were out of Standards.

Table 5. Goodness of Fit for Asejire Pollutants (Weibull).

Pollutants	(α; β)	sig.(α)	p-value	Rejection
Turbidity	4.141, 2.965	0.05	1.20e-6	Yes
Color	9.200, 5.235	0.05	0.000	Yes
pH	13.484, 7.752	0.05	0.004	Yes
DO	7.276, 8.008	0.05	0.000	Yes
Alkalinity	3.922, 42.328	0.05	5.59e-8	Yes
TH	5.687, 61.325	0.05	0.002	Yes
CaH	4.020, 42.508	0.05	0.110	**No**
Chloride	3.561, 21.527	0.05	0.004	Yes
Iron	16.981, 0.250	0.05	0.027	Yes
Silica	4.458, 11.346	0.05	0.071	**No**
Sol	5.534, 149.611	0.05	5.2e-4	Yes
DS	4.412, 119.557	0.05	9.07e-5	Yes
SS	2.791, 44.598	0.05	1.17e-8	Yes

It is established that the Weibull distribution is appropriate for the CaH level and Silica in Asejire at 5% level of significance.

Table 6. Goodness of Fit for Asejire Pollutants (Lognormal).

Pollutants	(μ, σ)	sig. (α)	p-value	Rejection
Turbidity	0.928, 0.368	0.05	5.16e-11	Yes
Color	1.613, 0.078	0.05	0.000	Yes
pH	2.011, 0.070	0.05	0.256	**No**
DO	2.022, 0.106	0.05	0.000	Yes
Alkalinity	3.629, 0.225	0.05	0.024	Yes
TH	4.027, 0.184	0.05	0.006	Yes
CaH	3.608, 0.338	0.05	0.0059	Yes
Chloride	2.931, 0.267	0.05	0.076	**No**
Iron	-1.418, 0.067	0.05	0.0649	**No**
Silica	2.293, 0.308	0.05	0.071	**No**
Sol	4.916, 0.188	0.05	0.0059	Yes
DS	4.663, 0.267	0.05	6.26e-5	Yes
SS	3.623, 0.347	0.05	1.9e-4	Yes

It is shown from the Table 6 that the lognormal distribution is appropriate for the pH, Chloride, Iron and Silica in Asejire at 5% level of significance.

Table 7. Goodness of fit for Eleyele Pollutants (Weibull).

Pollutants	(α; β)	sig. (α)	p-value	Rejection
Turbidity	1.620, 9.260	0.05	0.0001	Yes
Color	1.986, 8.651	0.05	0.0021	Yes
pH	22.199, 7.362	0.05	0.024	Yes
DO	1.469, 12.581	0.05	0.000	Yes
Alkalinity	2.941, 53.995	0.05	0.000	Yes
TH	9.446, 95.179	0.05	0.002	Yes
CaH	5.489, 70.462	0.05	0.001	Yes
Chloride	5.742, 38.763	0.05	2.02e-10	Yes
Iron	15.498, 2.414	0.05	0.000	Yes
Silica	6.559, 13.836	0.05	0.08	**No**
Sol	7.119, 259.99	0.05	0.000	Yes
DS	8.707, 180.640	0.05	0.000	Yes
SS	8.302, 74.070	0.05	1.123e-7	Yes

It is indicated from the Table 7 that the Weibull distribution is appropriate for the silica concentration in Eleyele at 5% level of significance.

Table 8. Goodness of fit for Eleyele Pollutants (Lognormal).

Pollutants	**(α; β)**	**sig. (α)**	**p-value**	**Rejection**
Turbidity	1.911, 0.639	0.05	0.054	**No**
Color	1.931, 0.408	0.05	0.0016	Yes
pH	1.971, 0.054	0.05	0.029	Yes
DO	1.469, 0.460	0.05	0.000	Yes
Alkalinity	3.809, 0.404	0.05	0.000	Yes
TH	4.498, 0.123	0.05	0.000	Yes
CaH	4.165, 0.187	0.05	0.001	Yes
Chloride	3.578, 0.151	0.05	5e-5	Yes
Iron	0.844, 0.076	0.05	0.0016	Yes
Silica	2.529, 0.256	0.05	0.001	Yes
Sol	5.502, 0.108	0.05	0.000	Yes
DS	5.152, 0.080	0.05	0.000	Yes
SS	4.233, 0.181	0.05	0.000	Yes

The Table 8 affirmed that the lognormal distribution is appropriate for only the Turbidity in Eleyele at 5% level of significance.

IV. Concluding Remarks

In this research, both rivers are polluted with at least one Pollutant. Asejire River is impure based on pH level than Eleyele River while Eleyele River is impure based on Chloride and Dissolved Solids concentration. Generally, Eleyele River is more polluted and unsafe than Asejire River as it is turbid, coloured, and based on average has high concentration of Chlorine.

Lognormal distribution described the pattern of the various pollutants better than Weibull distribution through the goodness of fit application. It is now recommended that Lognormal Distribution is appropriate while modelling water pollutants when comparing to Weibull Distribution.

References

[1] Aladesanmi T.O., Oladipo O.G. and Ali G.A. (2013): Aquatic Environmental Contamination: The fate of Asejire Lake in South-Western Nigeria. African Journal of Environmental Science and Technology. 7(6): 482-489.
[2] Arojojoye O.A. and Adeosun A.M. (2016): Effect of Environmental Pollution on Oxidative Stress Biomarkers in African Cat Fish (Clarias gariepinus) from Asejire River in Oyo State, Nigeria. Journal of Environmental and Occupational Science. 5(4):71-76.
[3] Asibor G., Edjere O., Adeniyi F. and Ogundele O. (2015). Using EF, PLI, and Igeo for the Assessment of Heavy Metal Pollution and Sediment Quality of Asejire Reservoir, South West Nigeria. International Journal of Environment and Pollution Research. 3(4): 77-90.
[4] Ipinmoroti M.O. (2013): Ichthyofauna diversity of Lake Asejire: Ecological implications. International Journal of Fisheries and Aquaculture 5(10): pp.248-252. http://www.academicjournals.org/IJFA.
[5] Lameed G.A. and Obadara P.G. (2006): Eco-Development Impact of Coca-Cola Industry on Biodiversity Resources at Asejire area, Ibadan, Nigeria. Journal of Fisheries International. (2 4):55-56.
[6] Obisesan K.O., Lawal M., Bamiduro T.A. and Adelakun A.A. (2013): Data Visualization and Change Point-Detection in Environmental Data: The case of Water Pollution in Oyo State. Journal of Scientific Research 12:181-190.
[7] W.H.O, (2006): Guidelines for Drinking-water Quality. Incorporating first.

AUTHOR'S PROFILE

Saheed A. Afolabi, He was born on 23rd December, 1990 in Osogbo town of Osun State, Nigeria. He finished his primary and secondary education in 2001 and 2007 respectively, he attended Federal Polytechnic Ede, Osun State between 2009 to 2011 to obtain his National Diploma in Statistics which he was the overall best student of the session, he did a Direct entry to the University of Ibadan, Ibadan, Oyo State, Nigeria to obtain BSc in Statistics, (2014) with First Class and MSc in Statistics, (2019) with a PhD grade and presently on his PhD programme. He is a member of Nigeria Statistical Association (2017), Professional Statisticians Society of Nigeria (2017), International Biometric Society (2018), Royal Statistical Society (2019) and American Statistical Association (2019). The Author's major fields are Envi--ronmental Statistics, Bayesian Inference and Mathematical Statistics but he has published papers in different fields.